

Lectures on Challenging Mathematics

Math Challenges 6

Number Sense

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1.7 Computational practices (part 1)

1. Zac has chosen five numbers from the set $\{1, 2, 3, 4, 5, 6, 7\}$. If he told Claudia what the product of the chosen numbers was, that would not be enough information for Claudia to figure out whether the sum of the chosen numbers was even or odd. What is the product of the chosen numbers?
2. A positive integer is written on each face of a cube. Each vertex is then assigned the product of the numbers written on the three faces intersecting the vertex. The sum of the numbers assigned to all the vertices is equal to 1001. Find the sum of the numbers written on the faces of the cube.
3. Four positive integers a, b, c, d satisfy

$$ab + a + b = 524,$$

$$bc + c + b = 146,$$

$$cd + c + d = 104.$$

Find all possible values of $a - d$.

4. Compute the number of ordered quadruples of integers (a, b, c, d) satisfying the following system of equations:

$$abc = 12000, \quad bcd = 24000, \quad cda = 36000.$$

5. Find the least positive integer n satisfying the following properties: n is divisible by 3 but not 9, and the sum of n and the product of the digits of n is divisible by 9.