

Lectures on Challenging Mathematics

Math Challenges 6

Counting

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Internal Use

1.9 Practices with binomial coefficients (part 3)

- Two identical decks of cards are shuffled together. What is the probability that the first 52 cards will contain all eight Aces?
- Consider a 3×3 point array. Five of the nine points will be colored in red, and the rest will be colored in blue.
 - Determine the total number of coloring schemes. (Two coloring schemes are considered different even though one can be obtained from the other via a rotation and/or a reflection.)
 - Some these coloring schemes satisfy the following property: There exist three blue points that are collinear and there does not exist three red points that are collinear. How many such coloring schemes are there?
- Find the smallest positive integer n for which the expansion of $(xy - 3x + 7y - 21)^n$, after like terms have been collected, has at least 1996 terms.
- A 5-tuple $(x_1, x_2, x_3, x_4, x_5)$ of nonnegative integers is called *magical* if exactly one of the five numbers is odd and the sum of these five numbers is equal to 99. How many magical 5-tuples are there?
- A license plate consists of 8 digits. It is called *even* if it contains an even number of 0's. Find the number of even license plates.