# Lectures on Challenging Mathematics

## Math Challenges 5

## Number Sense



Summer 2018

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## 1.9 Divisibility criteria and digits (part 2)

1. Consider the number $n = 10^{1002} - 4^{501}$.

   (a) What is the remainder of $5^k$, for each of $1 \le k \le 12$, divided by 8? Do you observe any pattern? If so, describe this pattern. Can you explain why this pattern works? Indeed, you may do so by either working on the last three digits of $5^k$ (that is; the remainder of $5^k$ divided by 1000) or factoring $5^k - 1$.

   (b) What is the remainder of $5^k$, for each of $1 \le k \le 12$, divided by 16? Do you observe any pattern? If so, describe this pattern. Can you explain why this pattern works? Indeed, you may do so by either working on the last four digits of $5^k$ (that is; the remainder of $5^k$ divided by 10000) or factoring $5^k - 1$.

   (c) What is the greatest power of 2 that is a factor of $n$?

2. A *palindrome*, such as 83438, is a number that remains the same when its digits are reversed. A palindrome between 1000 and 10000 is chosen at random. What is the probability that it is divisible by 7?

3. Several sets of prime numbers, such as $\{7, 83, 421, 659\}$, use each of the nine nonzero digits exactly once. What is the smallest possible sum such a set of primes could have? How many such sets of primes have their sums equal to this minimum possible sum?

4. Let $T = \{9^k \mid k \text{ is an integer}, 0 \le k \le 4000\}$. Given that $9^{4000}$ has 3817 digits and that its first (leftmost) digit is 9, how many elements of $T$ have 9 as their leftmost digit?

5. Determine whether there is any perfect square that ends in 10 distinct digits.