

Lectures on Challenging Mathematics

Math Challenges 5

Geometry

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1.4 Practices with geometric computations (part 2)

- Farmer Chong Gu glues together four equilateral triangles of side length 1 such that their edges coincide. He then drives in a stake at each vertex of the original triangles and puts a rubber band around all the stakes. Find the minimum possible length of the rubber band.
- Two distinct lines pass through the center of three concentric circles of radii 1, 2, and 3. In the right-hand side figure shown above, the total area of the closed regions labeled A through F is $8/13$ of the total area of unlabeled closed regions. What is the degree measure of the acute angle formed by the two lines?
- In rectangle $ABCD$, we have $AB = 8$, $BC = 9$. Point H and E lie on sides BC and AD , respectively, with $BH = 6$ and $DE = 4$. Line EC intersects line AH at G , and F is the foot of the perpendicular from G to line AD . Find GF .
- The incircle of triangle ABC touches the sides AB , BC , CA at F , D , E , respectively. Set $AB = c$, $BC = a$, $CA = b$. Express the length of each of the segments BD , DC , CE , EA , AF , FB in terms of a, b, c .
- Equilateral triangle XYZ is inscribed in a unit circle ω . Let W be a point other than X in the plane such that triangle WYZ is also equilateral. Determine the area of the region inside triangle WYZ that lies outside circle ω .

