# Lectures on Challenging Mathematics

## Math Challenges 4

## Number Sense





Winter 2018



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# Contents

## 1.7 Cubes and their sums and differences (part 2)

1. The number 1729 is known as the *Hardy-Ramanujan number* after a famous anecdote of the British mathematician G. H. Hardy regarding a visit to the hospital to see the Indian mathematician Srinivasa Ramanujan. The following is from *Quotations from Hardy*:

   > I remember once going to see him when he was ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavorable omen. "No," he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways."

   Given that one way is $1729 = 12^3 + 1^3$, find the other way. Use these two representations to factor 1729.

2. (Continuation) In memory of this incident, the least number which is the sum of two positive cubes in $n$ different ways is called the $n^{\text{th}}$ taxicab number. Hence 1729 is the $2^{\text{nd}}$ taxicab number, and it was first published by F. de Bessy in 1657. The $3^{\text{rd}}$ taxicab number, discovered by Leech in 1957, is

   $$87539319 = 167^3 + 436^3 = 228^3 + 423^3 = 255^3 + 414^3.$$

   Factor the $3^{\text{rd}}$ taxicab number. (This can be done with clever reasoning and estimation, *without* the assistance of any calculating device.)

3. Given that $5^x + 5^{-x} = 5$, compute $5^{3x} + 5^{-3x}$.

4. Find all pairs of integers $(a, b)$ such that $a^3 + b^3 = 91$.

5. Show that $\sqrt[3]{2+\sqrt{3}} + \sqrt[3]{2-\sqrt{3}}$ is a solution of the equation $x^3 - 3x - 4 = 0$.