Lectures on Challenging Mathematics

Math Challenges 4

Geometry

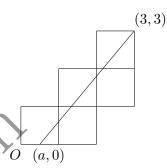Winter 2018

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Contents

1.6   Practices in geometry computations (part 1)

1. In triangle ABC, $AB = AC$. Points D and E lie on sides AB and AC respectively such that $AD = DE = EB = BC$. Find $\angle A$.

2. Five unit squares are arranged in the coordinate plane as shown, with the lower left corner at the origin O. The slanted line, extended from $(a, 0)$ to $(3, 3)$, divides the entire region into two regions of equal area. What is a?

3. Let $ABCD$ be a trapezoid with $AB \parallel CD$. Denote by M and N the midpoints of AB and CD. Suppose that $AD = 4$, $AB = 7$, $BC = 3$, and $CD = 12$. Find the length of the segment MN.

4. Consider a rectangle $ABCD$ with side lengths $AB = CD = 6$ and $BC = DA = 8$. Let X be a point on BD such that $AX \perp BD$. Find the length of each of following segments:

   (a) AX (b) BX (c) CX

5. Let $ABCD$ be a square. Points P and Q are variable points on segments AB and CD respectively such that the ratio between the areas of quadrilaterals $APQD$ and $BPQC$ is equal to $2 : 3$. Show that line PQ passes through a fixed point.