

Lectures on Challenging Mathematics

Math Challenges 4

Counting

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1.8 Seeing subtle differences (part 3)

1. There are twelve students in Mr. Fat geometry class. Mr. Fat wants his students to work on examples to support the famous result of Euler line; that is, the centroid, the orthocenter, and the circumcenter of a triangle lie on a line. Mr. Fat first chooses x students to form a group to find the centroid of a given triangle in the coordinate-plane, and then chooses another y students to find the orthocenter of the triangle. The remaining z students will find the circumcenter of the triangle. In how many ways can this be done if $(x, y, z) = (4, 4, 4)$? What if $(x, y, z) = (3, 4, 5)$?
2. There are 12 students in Mr. Fat geometry class. Mr. Fat has a group project on Pythagorean theorem. Mr. Fat first chooses x students to form a group to work on the project, and then chooses another y students to form another group to work on the project. The remaining z students form the last group to work on the project. In how many ways can this be done if $(x, y, z) = (4, 4, 4)$? What if $(x, y, z) = (3, 4, 5)$?
3. How many ways to split six students into (1) two groups of three? (2) three groups of two?
4. How many ways to split ten students into
 - (1) one group of one student, one group of two students, one group of three students, and one group of four students?
 - (2) two groups of five students?
 - (3) five groups of two students?
 - (4) two groups of four students and one group of two students?
 - (5) two groups of three students and one group of four students?
 - (6) two groups of three students and two groups of two students?
5. King Arthur and his twelve knights invited Robin Hood, James Bond, Harry Potter, Sir Bobby Charlton, and Sir Alex Ferguson for a dinner event. They are to going to sit around a round table of 18. King Arthur and Robin Hood sit next to each other so they can discuss politics. James Bond and Harry Potter sit together so they can talk about action movie. Sir Bobby Charlton and Sir Alex Ferguson sit together so they can chat about soccer and Manchester United. King Arthur and Harry Potter are to be sit opposite to each other. In how many ways can this be done? What if we further assume that King Arthur, James Bond, and Sir Bobby Charlton sit in clockwise order?