

Lectures on Challenging Mathematics

Integrated Mathematics 4

Geometry

Summer 2019

Zuming Feng

Phillips Exeter Academy and IDEA Math

zfeng@exeter.edu

©Copyright 2008 – 2019 Idea Math

Contents

1	Geometry	3
1.1	Revisiting congruence of triangles	3
1.2	Pythagorean theorem (part 1)	4
1.3	Parallelograms (part 1)	5
1.4	Revisiting Sentry theorem (part 1)	6
1.5	Parallelograms (part 2)	7
1.6	Revisiting Sentry theorem (part 2)	8
1.7	Special parallelograms (part 1)	9
1.8	Why there is no SSA congruence theorem? (part 1)	10
1.9	Special parallelograms (part 2)	11
1.10	Why there is no SSA congruence theorem? (part 2)	12
1.11	Trapezoids (part 1)	13
1.12	Geometry in the coordinate plane (part 1)	14
1.13	Trapezoids (part 2)	15
1.14	Angles and circles	16
1.15	Similarity of triangles	17
1.16	Circles and coordinate geometry	18
1.17	Similarity of triangles and quadrilaterals (part 1)	19
1.18	Pythagorean theorem (part 2)	20
1.19	Similarity of triangles and quadrilaterals (part 2)	21
1.20	Trapezoids (part 3)	22
1.21	Special elements in a triangle	23
1.22	Angles and geometric reasoning	24
1.23	Area and geometric reasoning	25
1.24	Geometry in the coordinate plane (part 2)	26
1.25	Areas and circles	27

1.13 Trapezoids (part 2)

1. Given an isosceles trapezoid with bases of 8 and 18 and an area of 156 square units, what is the number of units in the length of one of the non-parallel sides?
2. In a trapezoid $ABCD$, AB is parallel to CD , and $AB = 10$, $BC = 9$, $CD = 22$, and $DA = 15$. Points P and Q are marked on BC so that $BP = PQ = QC = 3$, and points R and S are marked on DA so that $DR = RS = SA = 5$. Find the lengths PS and QR .
3. A trapezoid has 11-inch and 25-inch parallel sides, and an area of 216 square inches.
 - (a) How far apart are the parallel sides?
 - (b) If one of the non-parallel sides is 13 inches long, how long is the other one? There are two answers to this question. It is best to make a separate diagram for each.

4. Accurate drawing is the theme of the problem. You should use your grid book well. You should choose the units of your graph wisely to avoid any unnecessary estimations.

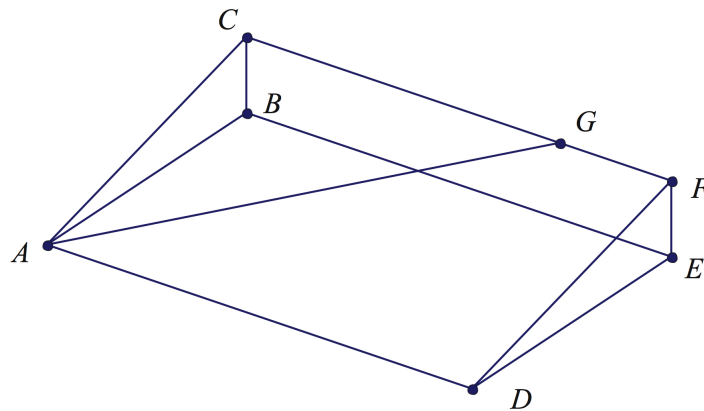
Draw a diagram of a non-isosceles trapezoid $ABCD$ with $AB \parallel CD$ and $MN = 9$, where M and N are the midpoints of AD and BC , respectively.

Is it possible that diagonals AC and BD divide the segment MN into three parts of lengths 4, 1, 4? If *not*, explain the reason; if *yes*, can you draw two accurate diagrams of two such trapezoids, one isosceles and one non-isosceles. From this information, is it possible to infer anything about the distance that separates the parallel sides? Explain.

5. (Continuation) Is it possible that diagonals AC and BD divide the segment MN into three parts of lengths 2, 3, 4? If *yes*, draw an accurate diagram of such a trapezoid; if *not*, explain the reason.

1.18 Pythagorean theorem (part 2)

- In triangle ABC , $\angle C = 90^\circ$. Medians are drawn from point A and point B in this right triangle to divide segment BC and AC in half, respectively. The lengths of the medians are 6 and $2\sqrt{11}$ units, respectively. How many units are in the length of segment AB ?
- Let DEF be a triangle and H the foot of the altitude from D to EF . If $DE = 60$, $DF = 35$, and $DH = 21$, what is the difference between the minimum and the maximum possible values for the area of DEF ?
- In triangle ABC , $AB = 8$, $\angle A = 75^\circ$ and $\angle B = 60^\circ$. Find the numerical values of the perimeter and the area of this triangle.
- Peyton's workout today is to run repeatedly up a steep grassy slope, represented by $ADFC$ in the diagram. The workout loop is $AGCA$, in which AG requires exertion and GCA is for recovery. Point G was chosen on the ridge CF to make the slope of the climb equal 20%.



Given that $ADEB$ and $BEFC$ are rectangles, ABC is a right angle, $AD = 240$, $DE = 150$, and $EF = 50$, find the distance from point G to point C .

- (Continuation) Peyton's next workout loop is $AHCA$, where H is a point on the path AG , chosen to make the slope of HC equal 20%. Find the ratio AH/AG , and explain your choice.