Lectures on Challenging Mathematics

Integrated Mathematics 4

Algebra

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Contents

1.13 Quadratic inequalities and quadratic graphs (part 1)

1. Sketch the graph of $y = -2x^2 + 7x - 3$. Solve the inequality $3 \geq 7x - 2x^2$.

2. When asked to solve the equation
$$\frac{x-2}{2x+3} > 1,$$
Alex again announced a plan: "Just solve for the product instead: $(x-2)(2x-3) > 1$". Blair puzzled: "Are you sure? I think you need to do something else first." Can you solve the inequality?

3. A small calculator company is doing a study to determine how to price one of its new products. The theory is that the income r from a product is a function of the market price p, and one of the managers has proposed that the quadratic model $r = p(3000 - 10p)$ provides a realistic approximation to this function.
   (a) What is the significance of the value $p = 300$ in this investigation?
   (b) Assume that this model is valid, and figure out the best price to charge for the calculator. How much income for the company will the sales for this calculator provide?

4. (Continuation) If the management is going to be satisfied as long as income from the new calculator is at least \$190,000, what is the range of prices p will be acceptable?

5. Solve the following inequalities.
   (a) $3x^2 + 1 \geq 5x$
   (b) $(2x-5)(x-3) > 3x(x-3)$
   (c) $(2x-1)(x+3) > (x+2)(x-6)$
   (d) $\dfrac{(x-1)(x+3)}{x-5} < x + 7$

1.20 Setting up (quadratic) relations (part 2)

1. The first part of a recent Outing Club hike was five miles uphill, and Byron found it slow going. The second part of the hike was six miles downhill, and the group was able to go 2 mph faster than the rate on the uphill section. The eleven-mile trip took a total of 3 hours and 20 minutes. What was Byron's hiking speed on the uphill section?

2. A bridge in the shape of a parabola spans a road that is 80 feet wide. The highest point of the bridge is 22 feet high. Determine whether a trailer that is 30 feet wide and 17 feet high can be moved under the bridge. Show your work and explain your method.

3. Solve the following inequalities.

   (a) $(x+2)(3-x) < (1-2x)(x-2)$ (b) $\dfrac{2x^2-3}{x+5} - 5 > 2x$

4. The driver of a red sports car, moving at r feet per second, sees a pedestrian step out into the road. Let d be the number of feet that the car travels, from the moment when the driver *sees* the danger until the car has been brought to a complete stop. The equation $d = 0.75r + 0.03r^2$ models the typical panic-stop relation between stopping distance and speed. It is based on data gathered in actual physical simulations. Use it for the following:

   (a) Moving the foot from the accelerator pedal to the brake pedal takes a typical driver three fourths of a second. What does the term $0.75r$ represent in the stopping-distance equation? The term $0.03r^2$ comes from physics; what must it represent?

   (b) How much distance is needed to bring a car from 30 miles per hour (which is equivalent to 44 feet per second) to a complete stop?

   (c) How much distance is needed to bring a car from 60 miles per hour to a complete stop?

   (d) Is it true that doubling the speed of the car doubles the distance needed to stop it?

5. (Continuation) At the scene of a crash, an officer observed that a car had hit a wall 150 feet after the brakes were applied. The driver insisted that the speed limit of 45 mph had not been broken. What do you think of this evidence?