Lectures on Challenging Mathematics

Integrated Mathematics 2

Algebra (Part 2)

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Contents

1.14 Exponential operations (part 3)

1. Verify that $(-8)^{\frac{1}{3}}$ can be evaluated, but that $(-8)^{\frac{1}{4}}$ cannot, and explain why $(-8)^{\frac{2}{6}}$ is ambiguous. To avoid difficulties like these, it is customary to restrict the base of an exponential expression to be a positive number when the exponent is not an integer.

2. Explain why calculating $z^{3.5}$ is a square-root problem. What does $m^{0.3}$ mean? Rewrite each of the following equation so that it has the form "$x = \ldots$".

   (a) $x^5 = a^3$ (b) $x^{\frac{1}{5}} = a^3$ (c) $x^{-\frac{2}{5}} = a^{\frac{3}{2}}$

   (d) $(2+x)^3 = m^{\frac{2}{5}}$ (e) $(x-1)^{-3.5} = n^{1.5}$ (f) $(1+x)^{15.6} = 2$

3. In order for a \$25,000 investment to double in ten years, what must be the annual rate of interest? Under the same interest rate, how long does it take to double a \$36,000 investment? To quadruple the investment?

4. Arrange the following numbers in increasing order from left to right.

$$2\sqrt{3}, \quad 3\sqrt{2}, \quad \sqrt{32}-\sqrt{8}, \quad \left(\sqrt{27}-\sqrt{3}\right)^{\frac{1}{2}}, \quad \left(\sqrt{18}+\sqrt{2}\right)^{\frac{1}{2}},$$

5. Solve the following equations by hand.

   (a) $8^x = 32$ (b) $4^{2016} - 4^{2015} - 4^{2014} + 4^{2013} = 90\left(4^x\right)$

1.20 Setting up (quadratic) relations (part 2)

1. The first part of a recent Outing Club hike was five miles uphill, and Byron found it slow going. The second part of the hike was six miles downhill, and the group was able to go 2 mph faster than the rate on the uphill section. The eleven-mile round trip took a total of 3 hours and 20 minutes. What was Byron's hiking speed on the uphill section?

2. A bridge in the shape of a parabola spans a road that is 80 feet wide. The highest point of the bridge is 22 feet high. Determine whether a trailer that is 30 feet wide and 17 feet high can be moved under the bridge. Show your work and explain your method.

3. Solve the following inequalities.

   (a) $(x+2)(3-x) < (1-2x)(x-2)$ (b) $\dfrac{2x^2-3}{x+5} - 5 > 2x$

4. The driver of a red sports car, moving at r feet per second, sees a pedestrian step out into the road. Let d be the number of feet that the car travels, from the moment when the driver *sees* the danger until the car has been brought to a complete stop. The equation $d = 0.75r + 0.03r^2$ models the typical panic-stop relation between stopping distance and speed. It is based on data gathered in actual physical simulations. Use it for the following:

   (a) Moving the foot from the accelerator pedal to the brake pedal takes a typical driver three fourths of a second. What does the term $0.75r$ represent in the stopping-distance equation? The term $0.03r^2$ comes from physics; what must it represent?

   (b) How much distance is needed to bring a car from 30 miles per hour (which is equivalent to 44 feet per second) to a complete stop?

   (c) How much distance is needed to bring a car from 60 miles per hour to a complete stop?

   (d) Is it true that doubling the speed of the car doubles the distance needed to stop it?

5. (Continuation) At the scene of a crash, an officer observed that a car had hit a wall 150 feet after the brakes were applied. The driver insisted that the speed limit of 45 mph had not been broken. What do you think of this evidence?