

1.6 Function Operations and Composition of Functions

• Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations (addition, subtraction, multiplication, and division) in f and g .

Addition $h(x) = f(x) + g(x)$

Subtraction $h(x) = f(x) - g(x)$

Multiplication $h(x) = f(x) \cdot g(x)$

Division $h(x) = \frac{f(x)}{g(x)}$

The domain of h consists of the x -values that are in the domains of both f and g . Additionally, the domain of a quotient does not include x -values for which $g(x) = 0$.

• The *composition* of the function f with the function g is: $h(x) = f(g(x))$

The domain of h is the set of all x -values such that x is in the domain of g and $g(x)$ is in the domain of f .

Problems:

1. Perform the indicated operation and state the domain.

(a) $f + g$; $f(x) = x + 5$, $g(x) = 3x - 1$ (b) $f - g$; $f(x) = x^2 - 4$, $g(x) = 2|x| + 1$

(c) $f \cdot g$; $f(x) = 9x - 4$, $g(x) = x + 3$ (d) $\frac{f}{g}$; $f(x) = 5x^2 - 2x$, $g(x) = x$

2. Let $f(x) = 10x - 3$, and $g(x) = x + 4$. Perform the indicated operation and state the domain.

(a) $f(g(x))$ (b) $g(f(x))$

(c) $f(f(x))$ (d) $g(g(x))$

3. Suppose $f(x) = 3x + 8$ and $g(x) = \sqrt{x - 4}$.

(a) Find $f(g(4))$ and $g(f(4))$; (b) Is $g(f(-4))$ defined? Why or why not?

4. Let $f(x) = 2x - 3$. Find $f(f(x))$ and $f(f(f(x)))$.

5. Let $f(x) = 3x^2 - 7$ and $g(x) = 2x + 5$. What is the absolute difference between $f(g(-2))$ and $g(f(-2))$?

6. If $f(x) = x + 2$ and $g(x) = x^2$, then for what value of x does $f(g(x)) = g(f(x))$? Express your answer as a common fraction.

Homework

1. Let $f(x) = 3x - 4$ and $g(x) = x + 5$. Perform the indicated operation.

(a) $f(x) + g(x)$

(b) $f(x) - g(x)$

(c) $f(x) \cdot g(x)$

(d) $\frac{f(x)}{g(x)}$

(e) $f(g(x))$

(f) $g(f(x))$

2. Let f and g be functions as defined below. What is $g(f(1))$ and $g(f(-2))$?

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0; \\ 4 - x & \text{if } x \geq 0, \end{cases} \text{ and } g(x) = \begin{cases} 2x & \text{if } x < 5; \\ 3x & \text{if } x \geq 5. \end{cases}$$

3. What is the value of $f(f(f(19) + 1) + 1)$ if $f(x) = \begin{cases} x^2 & \text{if } x \text{ is even} \\ \frac{x-3}{2} & \text{if } x \text{ is odd} \end{cases}$?