Lectures on Challenging Mathematics

Integrated Math 7

Algebra

Idea Math

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1.12 Linear parametric equations (part 5)

1. Find the point of intersection of the lines

   (a) $(-1+3t, 3+2t)$ and $(4-r, 1+2r)$ (b) $(2+3u, 5+7u)$ and $(11-13u, 17+19u)$

2. Graph the line that is described parametrically by $(x, y) = (2t, 5-t)$, then

   (a) confirm that the point corresponding to $t = 0$ is exactly 5 units from $(3, 9)$;

   (b) write a formula in terms of t for the distance from $(3, 9)$ to $(2t, 5-t)$;

   (c) find the other point on the line that is 5 units from $(3, 9)$;

   (d) find the point on the line that minimizes the distance to $(3, 9)$. There are multiple approaches to answer this question. First, answer the question by minimizing a quadratic expression. Second, explain why the resulting answer can be easily predicted.

3. Brett and Jordan are out driving in the coordinate plane, each on a separate straight road. The equations $B_t = (-3, 4) + t[1, 2]$ and $J_t = (5, 2) + t[-1, 1]$ describe their respective travels, where t is the number of minutes after noon.

   (a) Make a sketch of the two roads, with arrows to indicate direction of travel.

   (b) Where do the two roads intersect?

   (c) How fast is Brett going? How fast is Jordan going?

   (d) Do they collide? If not, who gets to the intersection first?

4. A car traveling east at 45 miles per hour passes a certain intersection at 3 pm. Another car traveling north at 60 miles per hour passes the same intersection 25 minutes later. To the nearest minute, figure out when the cars are exactly 40 miles apart.

5. Bug Fat starts at the point $A = (0, 1)$ and crawls, at a consistent speed of $\sqrt{13}$ units per second, perpendicularly towards to the line $\ell : 2x + 3y = 6$. Write a parametric equation which describes the position of the bug. How long does it take for the bug to reach ℓ and how long does it take for the bug to reach a point B that is symmetric to A across ℓ? Note that B is the *reflection* of A across the line ℓ. What are the coordinates of B?

   What if $A = (4, -5)$?

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1.17 Analytic transformations (part 2)

1. Apply the transformation $\mathcal{T}(x,y) = (2x+3y, -x+y)$ to the unit square, whose vertices are $(0,0)$, $(1,0)$, $(0,1)$, and $(1,1)$. Even though $\mathcal{T}$ is not a reflection, it is customary to call the resulting figure the *image* of the square. What kind of figure is it? Is this transfromation an isometry?

2. Decide whether each of the following transformation is an isometry, and give your reasons.

   (a) $\mathcal{T}_1(x,y) = \left(2x, \dfrac{1}{2}y\right)$ (b) $\mathcal{T}_2(x,y) = (-x, y+2)$

3. Given the transformation $\mathcal{F}(x,y) = (-0.6x - 0.8y, 0.8x - 0.6y)$, Shane calculated the image of the isosceles right triangle formed by $S = (0,0)$, $H = (0,-5)$, and $A = (5,0)$, and declared that $\mathcal{F}$ is a reflection. Morgan instead calculated the image of the *scalene* triangle formed by $M = (7,4)$, $O = (0,0)$, and $R = (7,1)$, and concluded that $\mathcal{F}$ is a rotation. Who was correct? Explain your choice, and account for the disagreement.

4. Determine if $\mathcal{F}$ is an isometry.

5. Explain why an isometry always transforms a right triangle onto a right triangle. In particular, does isometry preserve angle measures? If so, why this information is not part of the definition?

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1.30 Lattice points (part 5)

1. In triangle ABC, $A = (0, 0)$, $B = (4, 3)$, $C = (5, 12)$. Point D lies on side BC such that $\angle BAD = \angle CAD$. Find the coordinates of D.

2. Let $P = (-15, 0)$, $Q = (5, 0)$, $R = (8, 21)$, and $S = (0, 15)$. Draw quadrilateral $PQRS$ and measure its sides and angles. Is there anything remarkable about this figure? Make a conjecture from your observation and use vector operations or lattice points wise to confirm your conjecture.

3. Describe a transformation that carries the triangle with vertices $K = (0, 0)$, $L = (10, 5)$, and $M = (6, 18)$ onto the triangle with vertices $R = (3, 4)$, $P = (-2, 14)$, and $Q = (9, 22)$. Where does your transformation send $(5, 3)$?

4. Suppose that a quadrilateral is measured and found to have one of the following set of properties. Is this evidence enough to conclude that the quadrilateral is a parallelogram? Explain.

   (a) two pairs of equal nonadjacent sides

   (b) two pairs of equal nonadjacent angles

   (c) a pair of equal nonadjacent sides and a pair of equal nonadjacent angles

5. In a coordinate plane, determine if it is possible for an equilateral triangle to have all its vertices to be lattice points.

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