

# Lectures on Challenging Mathematics

## Math Challenges 3

### Geometry

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Zuming Feng

Phillips Exeter Academy and IDEA Math

zfeng@exeter.edu

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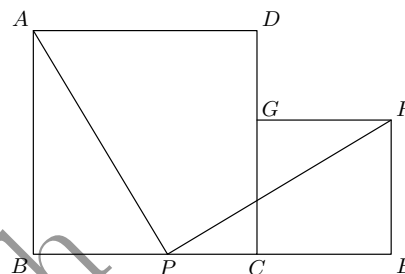
# Contents

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<b>1</b>	<b>Geometry</b>	<b>3</b>
1.1	Pythagorean theorem (part 1) . . . . .	3
1.2	Triangle congruence . . . . .	4
1.3	Special quadrilaterals (part 1) . . . . .	5
1.4	Pythagorean theorem (part 2) . . . . .	7
1.5	Special quadrilaterals (part 2) . . . . .	8
1.6	2-D and 3-D vision (part 1) . . . . .	9
1.7	Similarity of triangles . . . . .	10
1.8	2-D and 3-D vision (part 2) . . . . .	11
1.9	Trapezoids . . . . .	12
1.10	Geometric computations (part 1) . . . . .	13
<b>2</b>	<b>Geometry Supplement</b>	<b>15</b>
2.1	Revisiting regular polygons . . . . .	15
2.2	Triangle inequality . . . . .	16
2.3	Geometric computations (part 2) . . . . .	17
2.4	Practices in geometric computations (part 1) . . . . .	18
2.5	Area and circles . . . . .	19

### 1.4 Pythagorean theorem (part 2)

1. A  $5 \times 5$  square  $ABCD$  and a  $3 \times 3$  square  $CEFG$  are placed side to side. These two squares can be cut into pieces that will fit together to form a third square.



- (a) Find the length of a side of the third square.
- (b) Mark  $P$  on segment  $BC$  so that  $PB = 3$ , then draw segments  $PA$  and  $PF$ . Identify all pairs of congruent triangles in the resulting figure.
- (c) Segments  $PA$  and  $PF$  divide the squares into pieces. Arrange the pieces to form the third square.

2. (Continuation) For two arbitrary squares, will the preceding method always produce pieces that form a new square? When you rearrange these pieces to form the third square, explain why these pieces fit with each other well so there is no crease or overlapping.

3. (Continuation) Explain why this method leads to a *proof* of the Pythagorean theorem.

4. Convex quadrilateral  $ABCD$  has perpendicular diagonals. If  $AB = 25$ ,  $BC = 39$  and  $CD = 60$ , what is the length of segment  $DA$ ?

5. The square shown in the left-hand side diagram has been dissected into two trapezoids and two right triangles. These four pieces have been reassembled into the the rectangle shown in right-hand side diagram. Explain why there must be a crease in the rectangle? Why can't we see this crease? Describe this crease.

