

Lectures on Challenging Mathematics

Math Challenges 3

Counting

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1.6 Dealing with overcounting

1. Helen must read five books for her literature course. She may read any one of three biographies, any two of four mysteries, and any two of five science fiction books on her list. How many different sets of five books can she choose?
2. Find the number of distinguishable permutations of the letter in the word MISSISSIPPI.
3. An $11 \times 11 \times 11$ wooden cube is formed by gluing together 11^3 unit cubes. What is the greatest number of unit cubes that can be seen from a single point?
4. Before Rick can open his gym locker, he must recall the combination of three numbers. Two of the numbers are 17 and 24, but he has forgotten the third, and does not know the order of the numbers. There are 40 possibilities for the third number (from 0 to 39). At ten seconds per try, at most how long will it take him to exhaust all possibilities?
5. How many three-digit positive integers have the property that exactly two of the integer's digits are equal? How many of them are even?

