

Lectures on Challenging Mathematics

Math Challenges 3

Algebra

Winter 2018

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Internal Use

1.12 Revisiting work, distance and motion

1. Starting at 11:44 AM, Tony walked for 5 miles. He noticed that his average speed, expressed in minutes and seconds per mile, was numerically equal to the time at which he stopped if he thought of the hour as minutes and the minutes as seconds. Compute the time at which he stopped.
2. Bricklayer Brenda would take 9 hours to build a chimney alone, and bricklayer Brandon would take 10 hours to build it alone. When they work together, they talk a lot, and their combined output is decreased by 10 bricks per hour. Working together, they build the chimney in 5 hours. How many bricks are in the chimney?
3. Jon and Steve ride their bicycles along a path that parallels two side-by-side train tracks running in the east/west direction. Jon rides east at 20 miles per hour, and Steve rides west at 20 miles per hour. Two trains of equal length, traveling in opposite directions at constant but different speeds, each pass the two riders. Each train takes exactly 1 minute to go past Jon. The westbound train takes 10 times as long as the eastbound train to go past Steve. Find the length of the train in feet.
4. Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?
5. Mr. Fat was swimming up stream along the Exeter River yesterday. He lost his swimming cap at point A , but only found out 30 minutes later. Turning around immediately, he caught the lost cap at point B which is 3 miles from A . If he swam at a constant speed, what was the speed of the stream?