

Lectures on Challenging Mathematics

Integrated Mathematics 2

Geometry (Part 1)

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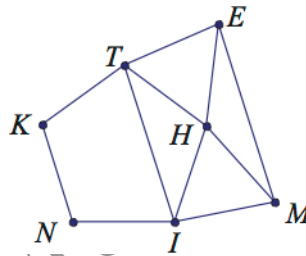
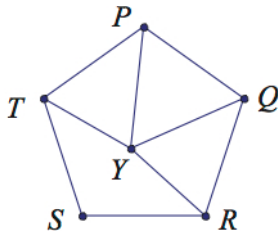
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1.14 Sentry theorem (part 2)

1. A regular, n -sided polygon has 18-degree exterior angles. Find the integer n .
2. Mark Y inside regular pentagon $PQRST$, so that PQY is equilateral. Are R, Y, T all on the same line? Explain.



3. Suppose that $ABCD$ is a square, and that CDP is an equilateral triangle, with P outside the square. What is the size of angle PAD ? What if P is inside the square?
4. In triangle ABC , $AB = AC$. Point D lies on side BC such that $AD = DB$ and $AC = CD$. Compute the angles of triangle ABC .
5. How many right triangles can be formed by connecting three of the eight vertices of a regular octagon?

1.19 Similarity of triangles (part 2)

1. *SSS Similarity theorem.* If the sides of two triangles are in proportion, then the two triangles are similar.

Triangles ABC and $A'B'C'$ are similar.

- (a) If AM and $A'M'$ are medians in the corresponding triangles, prove that triangles AMB and $A'M'B'$ are similar.
- (b) If AL and $A'L'$ are angle-bisectors in the corresponding triangles, prove that triangles ALB and $A'L'B'$ are similar.
- (c) If AK and $A'K'$ are altitudes in the corresponding triangles, prove that triangles AKB and $A'K'B'$ are similar.

2. A square is drawn on a piece of paper. Explain how to draw a square with area twice as large as that of the given square.

3. Draw a right triangle ABC with $\angle C = 90^\circ$ and altitude CD . List all similar triangles in your diagram. Find the length of segment CD and the side lengths of triangle ABC given that

- (a) $AD = 3$ and $BD = 4$;
- (b) $AD = 9$ and $AC = 12$.

4. Consider a quadrilateral $ABCD$ with $\angle B = \angle C = 90^\circ$, $AB = 6$, and $CD = 12$. Denote by P the intersection of the diagonals AC and BD and let Q be the foot of the perpendicular from P onto BC . Find the length of segment PQ .

5. Let AD , BE , CF be the altitudes in triangle ABC so that $AB = 13$, $AC = 15$, and $AD = 12$.

- (a) Find the lengths of the altitudes BE and CF .
- (b) Compute the lengths of segments AE and AF and show that triangles AEF and ABC are similar.