

Lectures on Challenging Mathematics

Math Challenges 2

Number Sense

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Internal Use

1.13 The decimal representation of numbers (part 2)

1. How many positive integers x are there such that $3x$ has 3 digits and $4x$ has four digits?
2. Find a four-digit perfect square whose first two digits are equal and whose last two digits are equal.
3. Call an integer *organized* if it is composed of distinct non-zero digits that are arranged in decreasing order from left to right. Find the greatest organized integer such that none of its digits divides the number itself.
4. Find all possible values of n such that the 2014-digit number $22\dots 2n$ (with 2013 2's followed by n as its units digit) is a multiple of $m = 8$. What if $m = 7$?
5. Find the largest six-digit number using each of the digits from 1 to 6 once such that this number is divisible by 6, deleting the 6 leaves a number divisible by 5, deleting the 5 leaves a number divisible by 4, and so forth down to 1. (Thus, for example, deleting the 6 from 136245 gives 13245, then deleting the 5 gives 1324, and so on.)