

# Lectures on Challenging Mathematics

## Math Challenges 2

### Geometry

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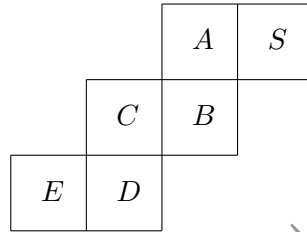
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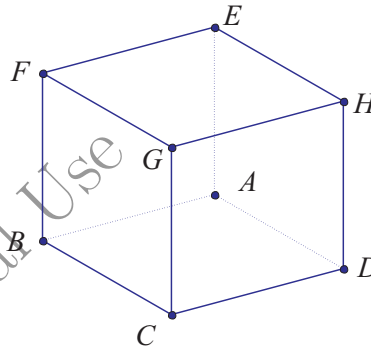
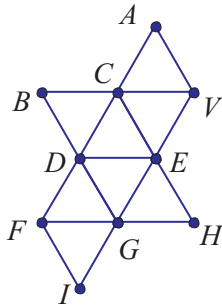
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## 1.7 3-D vision (part 1)

- When the net shown below is folded to form a cube, which face ( $A$ ,  $B$ ,  $C$ ,  $D$  or  $E$ ) will be the opposite the face labeled  $S$ ?



- A regular *octahedron* is a polyhedron composed of eight equilateral triangles, four of which meet at each vertex. How many vertices does a regular octahedron have? How many edges does a regular octahedron have? Is it possible to inscribe a regular octahedron into a cube so that each faces of the cube contains exactly one vertex of the octahedron?
- The net, in the left-hand side figure shown below, can be folded up to form an octahedron. When this is done, which two vertices are glued to vertex  $V$ ?



- In the right-hand side figure shown above, an ant is positioned at  $F$ , one of the eight vertices of a solid cube. It needs to crawl to vertex  $D$ , which is the furthest vertex from  $F$ , as fast as possible. Find one of the shortest routes. How many are there?

- A cube has 8 vertices, 12 edges, and 6 square faces. A soccer ball (also known as a *buckyball* or regular *truncated icosahedron*), given in the right-hand figure, has 12 pentagonal faces and 20 hexagonal faces. How many vertices and how many edges does a soccer ball have?

