# Lectures on Challenging Mathematics

# Math Challenges 2

# Counting

Winter 2021

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*"Cogito ergo Sum"* – *"I think, therefore I am"*

René Descartes (1596–1650)

*"Success is not final, failure is not fatal, it is the courage to continue that counts."*

Winston Churchill (1874–1965)

*"I can see that without being excited, mathematics can look pointless and cold. The beauty of mathematics only shows itself to more patient followers."*

Maryam Mirzakhani (1977–2017)

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## 1.3   Essential counting practices (part 3)

1. Mary has eight different T-shirts in the drawer and she decided to bring two to her trip to a national park. How many different sets of two shirts can Mary bring? What if she decides to bring three T-shirts?

2. On a 20-question true or false test, how many ways could you score exactly 90%?

3. With the outcome of an American football game at stake—5 points down with first and goal to go with 1 minute left—Coach Luzor uses his last timeout to make the arrangements for the next four possible plays. There are 6 pass plays and 4 running plays in the team's 1-minute offensive package.

    (a) Coach Luzor decides to choose 3 different pass plays and 1 running play. How many 4-play combinations does coach Luzor have?

    (b) For each 4-play combination in part (a), Coach Luzor decides to call the running play first and then the three pass plays in some order. How many play sequences there are?

    (c) Coach Luzor decides to call a running play first and then call 3 different pass plays next. How many possible 4-play sequences are there?

4. Five friends sat in a movie theater in a row containing 5 seats, numbered 1 to 5 from left to right. (The directions "left" and "right" are from the point of view of the people as they sit in the seats.) During the movie Ada went to the lobby to get some popcorn. When she returned, she found that Bea had moved two seats to the right, Ceci had moved one seat to the left, and Dee and Edie had switched seats, leaving an end seat for Ada. In which seat had Ada been sitting before she got up?

5. Tom Sawyer is painting a straight fence made of ten wooden stakes. He has three colors of paint: red, white, and blue. Each stake must be painted in one of the colors.

    (a) In how many ways can he paint the fence if there are no restrictions on its coloring?

    (b) What if he wants no two consecutive stakes to be the same color?

    (c) What if he wants every three consecutive stakes to be different colors?

## 1.9   Essential counting practices (part 9)

1. Seven boys and nine girls applied for jobs at a park. Two boys will be hired to be ticket takers, two girls will be chosen to be life guards and one other kid will be chosen to work at the concession stand. In how many ways can these five kids be chosen?

2. How many factors does 686000 have? Among these factors, how many are

   (a) odd numbers?                      (b) even numbers?

   (c) perfect squares?                   (d) perfect cubes?

3. What is the sum of the numbers less than 300 that have exactly nine divisors?

4. Each vertex of the following polygons is colored in a different color. How many distinct triangles can be formed by connecting three different vertices of a

   (a) pentagon?        (b) hexagon?        (c) heptagon?        (d) octagon?

5. What is the largest five-digit integer whose digits have a product equal to 5!? How many five-digit integers are there where the digits have a product equal to 5!?