

Lectures on Challenging Mathematics

Math Challenges 2

Algebra

Winter 2018

Edited by

Zuming Feng Yunhua Xu Chengde Feng Ivan Borsenco

©Copyright 2008 – 2018 Idea Math

Contents

©Copyright 2008 – 2018 Idea Math

I Algebra	3
1.1 Challenges with exponents and series (part 1)	3
1.2 Special types of linear systems	4
1.3 Challenges with exponents and series (part 2)	5
1.4 Linear inequalities in two variables	6
1.5 Proportions and mixed solutions	7
1.6 Polynomial multiplication and factorization	8
1.7 Word problems	9
1.8 Factorization of quadratic polynomials	10
1.9 Numbers and their pairwise sums	11
1.10 Revisiting quadratic functions	12

Internal Use

1.9 Numbers and their pairwise sums

1. Numbers a , b , c are labeled in that order from left to right on the number line. One tries to arrange numbers $a + b$, $b + c$, $c + a$ on the number line. Considering the relative positions of these three numbers, what are the possible scenarios?
2. Numbers a , b , c , d are labeled in that order from left to right on the number line. One tries to arrange numbers $a + b$, $b + c$, $c + d$, $a + c$, $b + d$, $a + d$ on the number line. Considering the relative positions of these six numbers, what are the possible scenarios?
3. A person from the audience thinks of four numbers. She tells the magician that their pairwise sums are

34, 36, 40, 42, 46, 48.

Can the magician find her numbers? What is the issue with this magic trick?

4. Five numbers are given. Their pairwise sums are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Find the sum of these numbers.
5. Ana, Barbara, and Carol want to know their average age. But no lady wants to disclose her age. Once they agree on a strategy they will fairly implement it. Is there a way to do that?