

# Lectures on Challenging Mathematics

## Math Challenges 2

### Algebra

Winter 2021

Zuming Feng  
Ivan Borsenco

*“Cogito ergo Sum” – “I think, therefore I am”*

René Descartes (1596–1650)

*“Success is not final, failure is not fatal, it is the courage to continue that counts.”*

Winston Churchill (1874–1965)

*“I can see that without being excited, mathematics can look pointless and cold. The beauty of mathematics only shows itself to more patient followers.”*

Maryam Mirzakhani (1977–2017)

# Contents

<b>1 Algebra</b>	<b>3</b>
1.1 Challenges with exponents and series (part 1)	3
1.2 Special types of linear systems	4
1.3 Challenges with exponents and series (part 2)	5
1.4 Linear inequalities in two variables	6
1.5 Word problems (part 1)	7
1.6 Polynomial multiplication and factorization	8
1.7 Word problems (part 2)	9
1.8 Factorization of quadratic polynomials	10
1.9 Numbers and their pairwise sums	11
1.10 Revisiting quadratic functions	12

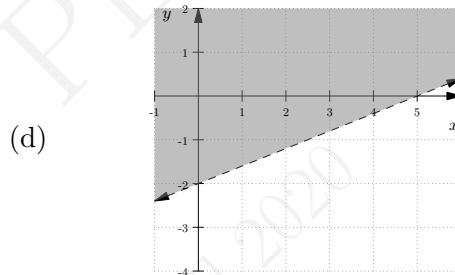
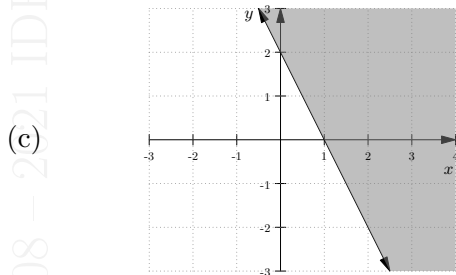
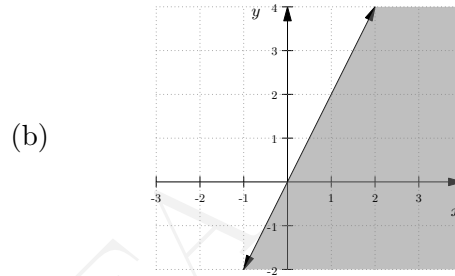
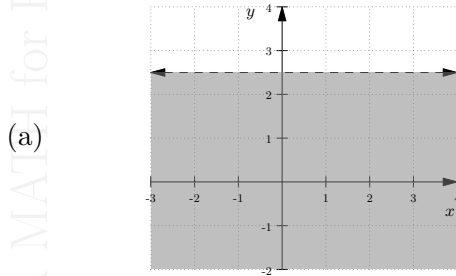
## 1.4 Linear inequalities in two variables

1. Check whether the ordered pair is a solution to  $2x - 3y \geq -2$ :

(a)  $(0, 0)$                       (b)  $(0, 1)$                       (c)  $(2, -1)$                       (d)  $(2, 2)$

Shade the region in the coordinate plane whose points satisfy the inequality.

2. Write inequalities whose solutions are shown in the graphs:



3. You are on a treasure-diving ship that is hunting for gold and silver coins. Objects collected by the divers are placed in a wire basket. One of the divers signals you to reel in the basket. It feels as if it contains no more than 50 pounds of material. If each gold coin weighs about 0.5 ounces and each silver coin weighs about 0.25 ounces, what are the different amounts of coins that could be in the basket?
4. The simultaneous conditions  $x - y < 6$ ,  $x + y < 6$ , and  $x > 0$  define a region  $\mathcal{R}$ . How many lattice points are contained in  $\mathcal{R}$ ?
5. Sketch the graph of the following inequalities:

(a)  $xy \geq 0$

(b)  $(x + y)(x - y) < 0$

## 1.7 Word problems (part 2)

- Halfway through the basketball season, Fran Tastik has made 24 out of 40 free throw shots.
  - What is Fran's average, expressed as a percent?
  - Fran anticipates getting 30 more free throw tries by the end of the season. How many these must Fran make, in order to have a season average that is at least 70%?
- Alden paid to have some programs printed for the football game last weekend. The printing cost per program was 54 cents, and the plan was to sell them for 75 cents each. Poor weather kept many fans away from the game, however, so unlucky Alden was left with 100 unsold copies, and lost \$12 on the venture. How many programs did Alden have printed?
- The cooling system of Alex's car holds 10 quarts. It is now filled with a mixture that is 60% water and 40% antifreeze. Hearing a weather forecast for severe cold, Alex decides to increase the strength of the antifreeze mixture to 50%. To do this, Alex must drain off a certain number of quarts from the cooling system and then replace them by pure antifreeze. How many quarts must be drained?
- An arithmetic sequence of positive integers has 8 terms. The sum of these 8 terms is equal to 2008. What is the smallest possible value of any member of the sequence?
- Dale's investment portfolio consisted of shares of internet stock and copper stock. During the year of 2017, the value of his copper shares decreased from \$ 20000 to \$ 19500. If the value of his copper shares are under \$14200, he will sell all his copper shares. If his copper shares continue to decrease by \$ 500 per year, when will he sell all his copper shares?

During the same year, the value of his internet shares increased by 20%, from \$ 10000 to \$ 12000. If the value of his internet shares keeps growing at an annual rate of 20%, what is the value of his internet share by the end of 2018? 2019? By the end of which year will the value of his internet shares exceed that of his copper shares? (Use a calculating device to deal with the calculations.)