

Lectures on Challenging Mathematics

Math Challenges 1

Number Sense

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1.10 Switching the order of operations (part 3)

1. Evaluate:

$$\frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} \quad \text{and} \quad \left(\frac{1}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{1}{3 \cdot 4 \cdot 5 \cdot 6} \right) \cdot 84.$$

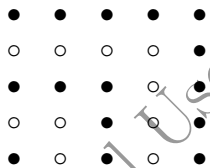
2. Given that m and n are positive integers such that

$$2 \times 2 \times 4 \times 4 \times 4 \times 4 \times \underbrace{8 \times 8 \times \cdots \times 8}_{8 \text{ 8s}} \times \underbrace{5 \times 5 \times \cdots \times 5}_{m \text{ 5s}} = \underbrace{10 \dots 0}_{n \text{ 0s}},$$

find m and n .

3. The product of all positive integer divisors of 72, including 1 and 72 itself, is equal to $2^k 3^\ell$. Find the integers k and ℓ .
4. Compute the sums:
- $1 + 3 + 5 + \cdots + 21$
 - $1 + 3 + 5 + \cdots + 63$
 - $1 + 3 + 5 + \cdots + 99$

What is common to all the answers that you get? Do you think this is a coincidence? The diagram below may be helpful in explaining your result.



5. Compute $(123 + 4)(123 + 5) - 123 \cdot 132$ and $(9876 + 4)(9876 + 5) - 9876 \cdot 9885$.