

# Lectures on Challenging Mathematics

## Math Challenges 1

### Geometry

Winter 2018

Zuming Feng

Phillips Exeter Academy and IDEA Math

zfeng@exeter.edu

©Copyright 2008 – 2018 Idea Math

# Contents

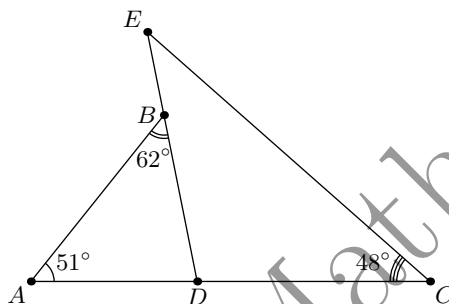
©Copyright 2008 – 2018 Idea Math

<b>1</b>	<b>Geometry</b>	<b>3</b>
1.1	Points, lines, and planes (part 1) . . . . .	3
1.2	Segments and rays (part 1) . . . . .	5
1.3	Points, lines, and planes (part 2) . . . . .	7
1.4	Segments and rays (part 2) . . . . .	9
1.5	Angles (part 1) . . . . .	10
1.6	Angles (part 2) . . . . .	12
1.7	Parallel lines (part 1) . . . . .	14
1.8	Parallel lines (part 2) . . . . .	16
1.9	Angles (part 3) . . . . .	17
1.10	Parallel lines (part 3) . . . . .	19
<b>2</b>	<b>Geometry Supplement</b>	<b>21</b>
2.1	3-D vision (part 1) . . . . .	21
2.2	3-D vision (part 2) . . . . .	23
2.3	3-D vision (part 3) . . . . .	24
2.4	Matchstick puzzles . . . . .	25

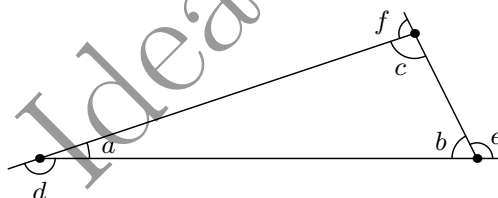
Idea Math  
Internal Use

### 1.9 Angles (part 3)

- In triangle  $ABD$ ,  $\angle BAD = 51^\circ$  and  $\angle ABD = 62^\circ$ . Let  $E$  be a point on the extension of  $DB$  through  $B$  and let  $C$  be a point on the extension of  $AD$  through  $D$ . Given that  $\angle ECD = 48^\circ$ , find the value of  $\angle DEC$ .

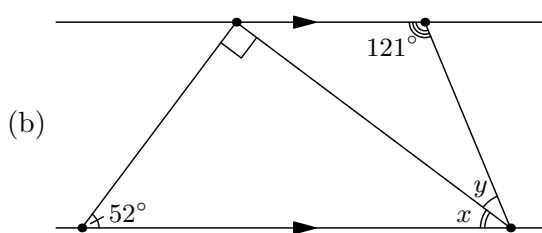
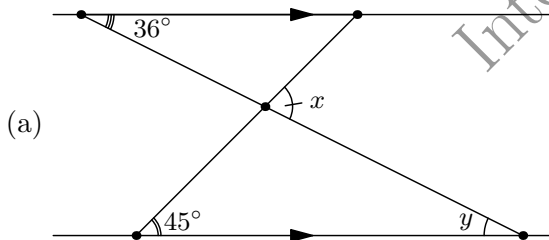


- Using the diagram, complete the following exercises:



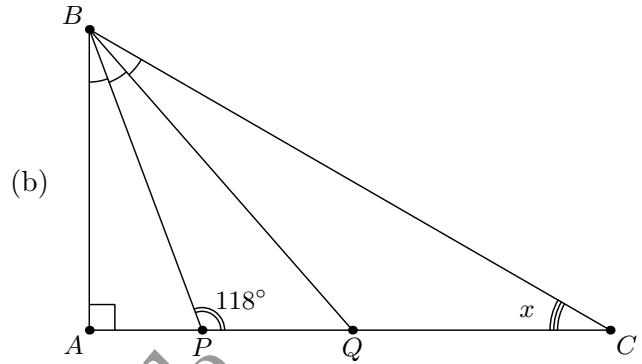
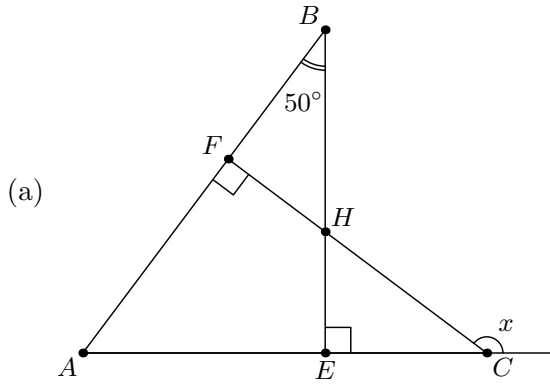
- $\angle a = 40^\circ$ ,  $\angle b = 60^\circ$ , find  $\angle f$ ;
- $\angle a = 43^\circ$ ,  $\angle c = 71^\circ$ , find  $\angle e$ ;
- $\angle d = 120^\circ$ ,  $\angle b = 30^\circ$ , find  $\angle c$ ;
- $\angle d = 143^\circ$ ,  $\angle e = 134^\circ$ , find  $\angle f$ .

- In each diagram find the values of  $x$  and  $y$ :



- Two lines are *perpendicular* if and only if they intersect at a right angle. If lines  $p$  and  $q$  are perpendicular, the notation  $p \perp q$  is used. Perpendicular lines are marked with a “square” (a box) drawn at the point of intersection.

Find the unknown value  $x$ . (Note: In part (b) lines  $BP$  and  $BQ$  trisect angle  $\angle ABC$ ).



5. The angle trisectors of  $\angle B$  and  $\angle C$  of triangle  $ABC$  intersect at points  $P$  and  $Q$  as shown in the diagram below. Denote  $\angle ABP = \angle PBQ = \angle QBC = x$  and  $\angle ACP = \angle PCQ = \angle QCB = y$ . Show that  $\angle BAC + \angle BQC = 2\angle BPC$ .

