

# Lectures on Challenging Mathematics

## Math Challenges 1

### Geometry

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*“Cogito ergo Sum” – “I think, therefore I am”*

René Descartes (1596–1650)

*“Success is not final, failure is not fatal, it is the courage to continue that counts.”*

Winston Churchill (1874–1965)

*“I can see that without being excited, mathematics can look pointless and cold. The beauty of mathematics only shows itself to more patient followers.”*

Maryam Mirzakhani (1977–2017)

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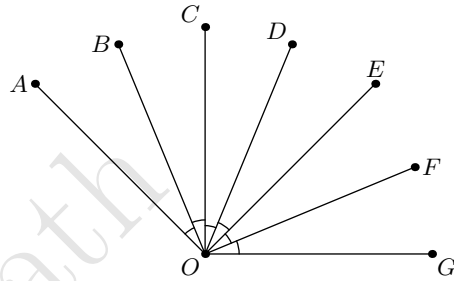
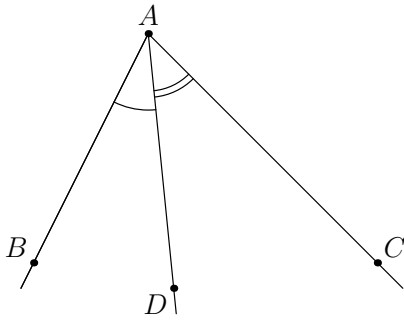
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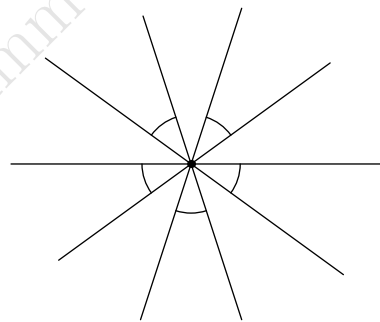
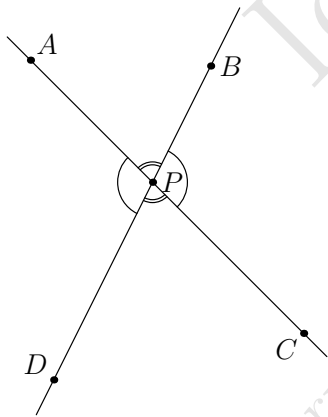
### 1.5 Angles (part 2)

1. *Adjacent* angles are two angles in a plane that have a common vertex and a common side but no common interior points. In the diagram below, angles  $\angle BAD$  and  $\angle CAD$  are adjacent.



Angle  $\angle AOG$  is divided by rays  $OB, OC, OD, OE, OF$  into six congruent angles. How many pairs of congruent adjacent angles are there?

2. *Vertical angles* is a pair of non-adjacent angles formed by the intersection of two straight lines. When two lines intersect they form two pairs of vertical angles. Vertical angles are congruent. For example, in the diagram below  $\angle APB = \angle CPD$  and  $\angle APD = \angle BPC$ .



Five lines intersect in one point. Given that five alternating non-adjacent angles are congruent, as shown in the diagram above, find the measure of each of them.

3. An angle of measure between  $0^\circ$  and  $90^\circ$  is called an *acute* angle. An angle of measure between  $90^\circ$  and  $180^\circ$  is called an *obtuse* angle.

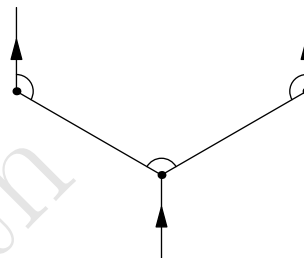
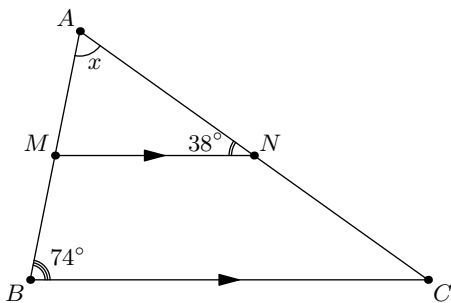
An acute angle  $\alpha$  and an obtuse angle  $\beta$  are prime numbers. If  $\alpha + \beta = 112^\circ$ , find all possible pairs  $(\alpha, \beta)$  that satisfy the conditions of the problem.

4. Ray  $OB$  is the angle-bisector of  $\angle AOC$ , whose measure is  $108^\circ$ . Ray  $OP$  is the angle-bisector of  $\angle AOB$  and ray  $OQ$  is the angle-bisector of  $\angle POC$ . Find the measure of  $\angle BOQ$ .

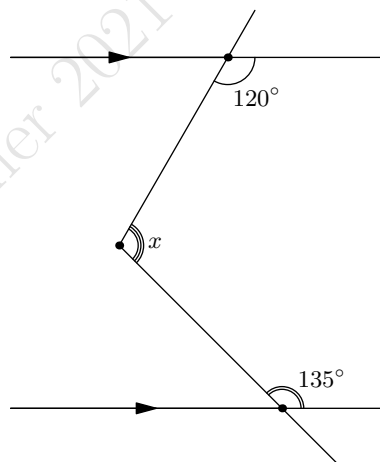
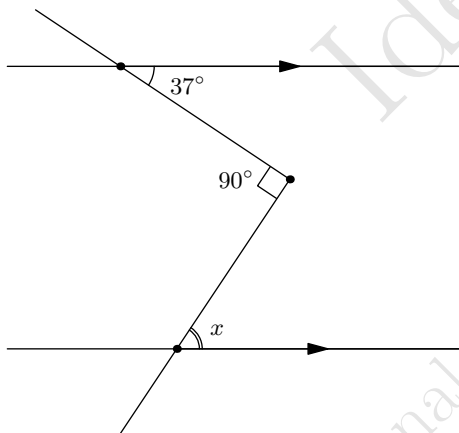
5. Consider a right angle  $\angle AOE$  and rays  $OB$ ,  $OC$ , and  $OD$  dividing this angle into four angles. It is given that  $OB$  is the angle-bisector of  $\angle AOC$  and  $OD$  is the angle-bisector of  $\angle COE$ .
- (a) Find  $\angle BOD$  assuming  $\angle AOC = 40^\circ$ .
  - (b) Find  $\angle BOD$  assuming  $\angle AOC = 58^\circ$ .
  - (c) Find  $\angle BOD$  choosing any size for  $\angle AOC$ .

## 1.8 Parallel lines (part 2)

1. Points  $M$  and  $N$  are lying on the sides  $AB$  and  $AC$  of triangle  $ABC$  such that  $MN$  is parallel to  $BC$ . Given that  $\angle B = 74^\circ$  and  $\angle ANM = 38^\circ$ , find  $\angle A$ .



2. In the diagram above we have three parallel rays and three congruent angles that are marked. Find the measure of each of these angles.
3. In each of the diagrams extend segments to find the unknown angle  $x$ .



4. A *trapezoid* is quadrilateral with *exactly* one pair of parallel sides. A trapezoid has a 35-degree angle and a 55-degree angle. What are the other angles? What if it has a 75-degree angle and a 105-degree angle?
5. A *parallelogram* is a quadrilateral that has two pairs of parallel sides. Explain why the opposite angles in a parallelogram are congruent.