

Lectures on Challenging Mathematics

Math Challenges 1

Counting

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“Cogito ergo Sum” – “I think, therefore I am”

René Descartes (1596–1650)

“Success is not final, failure is not fatal, it is the courage to continue that counts.”

Winston Churchill (1874–1965)

“I can see that without being excited, mathematics can look pointless and cold. The beauty of mathematics only shows itself to more patient followers.”

Maryam Mirzakhani (1977–2017)

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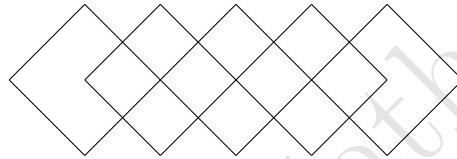
1.4 Basic counting practices (part 4)

1. A combination lock shown on the right has numbers from 0 through 39. This brand of lock must use three different numbers (in a specific order) in its combination. How many possible combinations exist for one of these locks?
2. How many ways can six DVDs (The Lion King, Finding Nemo, Up, Frozen, Madagascar, Cars) be lined up on a shelf?
3. (Continuation) Among all these lineups,
 - (a) how many have Frozen always to the immediate left of Cars?
 - (b) how many have Frozen always to the left of Cars?
4. A car holds exactly six people, but only two of those six can drive the car. What is the number of ways that the six people can be seated in the car on a ride?
5. The lockers at Math Middle School were numbered with consecutive multiples of 3 starting with 57. Students are assigned a locker at random. If there are 376 lockers in the school what is the chance that Pythagoras will be assigned a locker numbered more than 375?



1.8 Basic counting practices (part 8)

1. Katy and Mike are making candy bar packets for a Halloween party. Katy has 80 Kit Kat chocolate bars and she wants to put them into packets of 2 or 3. Mike has 100 Milky Way chocolate bars and he wants to put them into packets of 2 or 4. In how many ways can Katy and Mike make their packets?
2. How many rectangles of any size are in the diagram shown below?



3. When writing the positive integers from 1 to 1000 how many times is the digit 6 written?
4. Aunt Mary and uncle Sam are celebrating labor day weekend with Aaron, Baron, and Caron. Aunt Mary made six *indistinguishable* cupcakes to give to Andrew, Brad, and Cindy. In how many ways can she distribute these cupcakes among the kids so that each of them gets at least one cupcake?
(The term *indistinguishable* refers to the fact that the cupcakes are so identical that there is no way to tell them apart.)
5. (Continuation) Uncle Sam has six *distinguishable* balloons.
 - (a) If each child gets exactly one balloon (and uncle Sam keeps the remaining three), in how many different ways can this be done?
 - (b) If each child gets exactly two balloons, in how many different ways can this be done?