Lectures on Challenging Mathematics

Math Challenges 1

Algebra

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Edited by

Zuming Feng Yunhua Xu Chengde Feng Ivan Borsenco

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1.6 Algebraic expressions and operations (part 2)

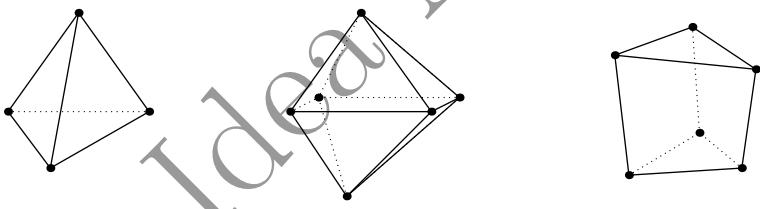
1. The perimeter of a rectangle is 100 and its length is x. What expression represents the width of the rectangle? What expression represents the area of the rectangle?

2. Without using parentheses, write an expression equivalent to $3(4(3x-6)-2(2x+1))$.

3. Given that $m = 25q + 10d + 5n + c$, find the value of m when $q = 3$, $d = 5$, $n = 7$, $c = 11$. Make up a word problem to go with the equation $25q + 10d + 5n + c = 100$.

4. Determine the number of quadruples (q, d, n, c) of positive integers such that
$$25q + 10d + 5n + c = 100.$$

5. For a 3-dimensional object, let v denote the number of its vertices, e denote the number of its edges, and f denote the number of its faces. For each of the following object, compute the value $v - e + f$. (For example, for a cube, we have $v = 8$, $e = 12$, $f = 6$, and $v - e + f = 2$.)



   (a) A tetrahedron. (Shown in the left-hand side figure above.)
   (b) An octahedron. (Shown in the middle figure above.)
   (c) A triangular prism. (Shown in the right-hand side figure above.)
   (d) The solid obtained in the following way: Gluing 27 unit cubes together to form a $3\times3\times3$ cube, and removing 8 unit cubes one from each corner.