

Lectures on Challenging Mathematics

Math Challenges 1

Algebra

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“Cogito ergo Sum” – “I think, therefore I am”

René Descartes (1596–1650)

“Success is not final, failure is not fatal, it is the courage to continue that counts.”

Winston Churchill (1874–1965)

“I can see that without being excited, mathematics can look pointless and cold. The beauty of mathematics only shows itself to more patient followers.”

Maryam Mirzakhani (1977–2017)

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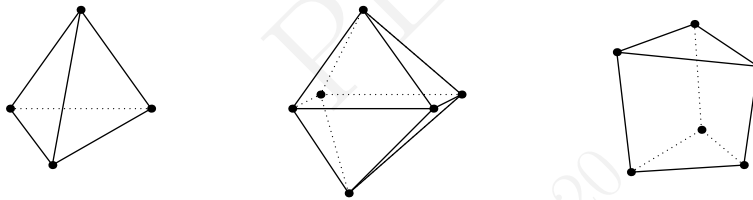
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1.6 Algebraic expressions and operations (part 2)

1. The perimeter of a rectangle is 100 and its length is x . What expression represents the width of the rectangle? What expression represents the area of the rectangle?
2. Without using parentheses, write an expression equivalent to $3(4(3x - 6) - 2(2x + 1))$.
3. Given that $m = 25q + 10d + 5n + c$, find the value of m when $q = 3$, $d = 5$, $n = 7$, $c = 11$. Make up a word problem to go with the equation $25q + 10d + 5n + c = 100$.
4. Determine the number of quadruples (q, d, n, c) of positive integers such that

$$25q + 10d + 5n + c = 100.$$

5. For a 3-dimensional object, let v denote the number of its vertices, e denote the number of its edges, and f denote the number of its faces. For each of the following object, compute the value $v - e + f$. (For example, for a cube, we have $v = 8$, $e = 12$, $f = 6$, and $v - e + f = 2$.)



- (a) A tetrahedron. (Shown in the left-hand side figure above.)
- (b) An octahedron. (Shown in the middle figure above.)
- (c) A triangular prism. (Shown in the right-hand side figure above.)
- (d) The solid obtained in the following way: Gluing 27 unit cubes together to form a $3 \times 3 \times 3$ cube, and removing 8 unit cubes one from each corner.

1.8 Revisiting units and percentage problems (part 1)

1. Darrell made 8 kg of trail mix for his family's hiking trip. His family ate 4600 g of the trail mix on the hiking trip. What percent of the mix was left for Darrell?
2. On 3 January 2004, after a journey of 300 million miles, the rover Spirit landed on Mars and began sending back information to Earth. It landed only six miles from its target. This accuracy is comparable to shooting an arrow at a target fifty meters away and missing the exact center by what distance, in millimeters? (Note that "Okay, so you're a rocket scientist That don't impress me much" is part of the lyrics of the hit song *That Don't Impress Me Much* by Shania Twain. After you see the result of this problem, are you impressed by rocket scientists' work?)
3. Many major-league baseball pitchers can throw the ball at 90 miles per hour. At that speed, how long does it take a pitch to travel from the pitcher's mound to home plate, a distance of 60 feet 6 inches? Give your answer to the nearest hundredth of a second.
4. A gas pump is steadily dispensing fuel. Madison holds her watch up to the price display and notice that at 8 : 48 : 10 (that is 8 : 48 and 10 seconds) the display reads \$4.50, while at 8 : 48 : 20 it reads \$11.90. If the pump shuts off at 8 : 49 : 05, then how much does Madison pay for the fuel that day?
5. When $10!$ seconds have elapsed, how many weeks have gone by?