Lectures on Challenging Mathematics

Integrated Mathematics 1

Algebra (Part 1)

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2.9 Units (part 1)

1. A runner plans to run 10000 meters, on a regular 400-meter track, in a world-class time of 27 minutes and 30 seconds. Running at a constant rate, what will the runners time be at the 4-lap mark (that is, completing 4 laps)?

2. The standard oil barrel of 42 US gallons is used in the United States as a measure of crude oil and other petroleum products. On September 24^{th} 2012, crude oil price is set at \$92.09 per barrel. On the same day, the US national average price for regular gasoline is \$3.805. One financial tracking institution reported that the profit-margins is about 29.6 cents per gallon for refining crude oil into gasoline. Estimate the cost, in cents per gallon, for refining crude oil into regular gasoline.

3. Anna and Becky both participate in a bicycle race of d miles. Anna pedals at 20 mph, Becky pedals at 18 mph, and Anna crosses the finish line nine minutes before Becky.

   (a) Explain the respective meaning of the expressions $\dfrac{d}{20}$ and $\dfrac{d}{18}$.

   (b) Explain the meaning of the expression $\dfrac{d}{18} - \dfrac{d}{20}$.

   (c) Explain why the equation $\dfrac{d}{18} - \dfrac{d}{20} = 9$ is false.

   (d) Find a correct equation in d and solve for d.

4. Jan has a $20'' \times 20'' \times 20''$ gift box that needs to be placed carefully into a $2' \times 2' \times 2'$ shipping carton, surrounded by packing peanuts. How many half-cubic-foot bags of peanuts does Jan need to buy?

5. A slow 24-hour clock loses 25 minutes a day. At noon on the first of October, it is set to show the correct time. When will be the next time this clock shows the correct time? If someone was living relying on the clock's time, how many days he believes have passed from the first October till that moment?

4.8 The slope of a line (part 3)

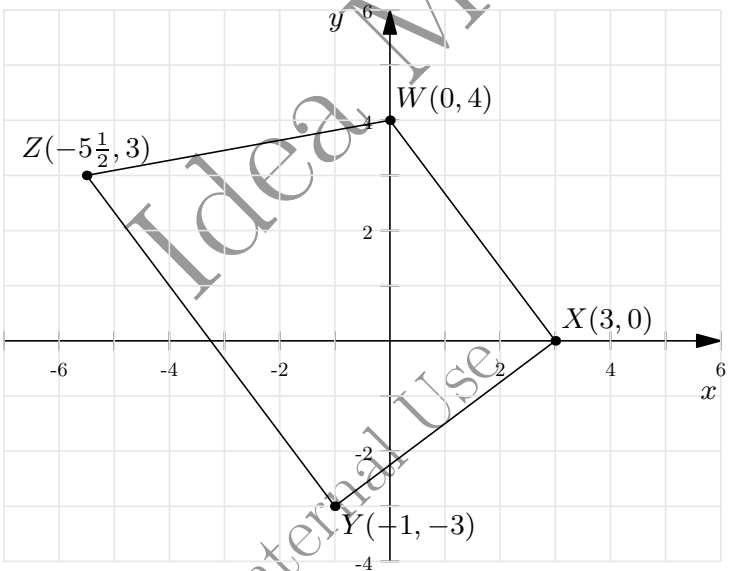
1. Find the equations of at least three lines that intersect each other at the point $(6, -2)$.

2. Find values for a and b that make $ax + by = 14$ parallel to $12 - 3y = 4x$. Is there more than one answer? If so, how are the different values for a and b related?

3. Find an equation for the line that passes through the point $(-3, 6)$, parallel to the line through the points $(0, -7)$ and $(4, -15)$. Write your answer in point-slope form.

4. Plot points $A = (-6, 1)$, $B = (-4, -3)$, and $C = (2, 0)$. Write equations in a point-slope form for the lines AB and BC. Note that the product of the slopes of these is equal to -1. What can be said about the lines AB and BC looking at the graph?

5. The coordinates of the vertices of quadrilateral $XYZW$ are shown in the diagram below.



(a) Find a pair of parallel and a pair perpendicular sides in the quadrilateral $XYZW$.

(b) Write in a point-slope form equations of the lines containing the two parallel sides. How do you know these lines are parallel?

(c) Write in a point-slope form equations of the lines containing the two perpendicular sides. How do you know these lines are perpendicular?

5.8 Rules for operations with integer exponents (part 4)

1. For integers m and n, we have been using the following rules of exponents:

   (i) $a^m \cdot a^n = a^{m+n}$ (ii) $\dfrac{a^m}{a^n} = a^{m-n}$ (iii) $a^m \cdot b^m = (ab)^m$ (iv) $\dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$

   Think of a pair of positive integers m and n, and a pair of a positive integer m and a negative integer n. Verify all of these rules using each pair.

2. Simplify the expressions:

   (a) $2^{11} \cdot 3^9 \cdot 6^{-7}$

   (b) $(3^{-7} + 3^{-7} + 3^{-7})(2^{-7} + 2^{-7})$

   (c) $\dfrac{(5^5)^4 \cdot 25^2}{125^7}$

   (d) $\left(2^3\right)^4 \cdot \left(4^3\right)^{-2}$

   (e) $\dfrac{(2^3)^4 \cdot (2^3)^4}{(4^3)^2 \cdot (2^4)^3}$

   (f) $\dfrac{91^{13} \cdot 13^{-13}}{7^{-7} \cdot 133^7}$

3. Determine a and b if $49^3 \cdot 343^6 \cdot 2401^a = 1$ and $4^b \cdot 8^{-3} \cdot 16^5 \cdot 32^{-6} = 2$.

4. A *geometric progression* or *geometric sequence* is a list in which each term is obtained by applying a constant multiplier to the preceding term. What is the 100^{th} term in each of the following geometric progressions?

   (a) $1, 2, 4, 8, \ldots$

   (b) $6, 18, 54, \ldots$

5. (Continuation) Compute the product of the first ten terms in each sequence.