

Lectures on Challenging Mathematics

Math Olympiads

Geometry

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1.13 Angle chasing and cyclic quadrilaterals (part 3)

1. [New Problems in Euclidean Geometry, by David Monk] Let $ABCD$ be a cyclic quadrilateral. The perpendicular bisector of segment CD meets lines AD and BD at P and Q , respectively. Prove that $\angle ACQ = \angle PCB$.
2. Let $ABCD$ be a cyclic quadrilateral with $BC = BA + CD$. Prove that the bisectors of angles A and D intersect on segment BC .
3. [New Problems in Euclidean Geometry, by David Monk] Quadrilateral $ABCD$ is inscribed in circle ω . Diagonals AC and BD meet at R , and rays AD and BC meet at P . Let X, Y, Z be the feet of the perpendiculars from D to lines AC, BC, PR , respectively. Prove that the circumcircle of XYZ passes through the midpoint of segment CD .
4. [New Problems in Euclidean Geometry, by David Monk] In triangle ABC , $AB \neq AC$. Point D is the foot of the perpendicular from A to line BC . Points M and N are the midpoints of segments BC and AD , respectively. Point P is the foot of the perpendicular from B to line AM . Prove that line MN is tangent to the circumcircle of CMP .
5. Let ABC be an acute triangle, and let AA_1, BB_1 , and CC_1 be its altitudes. Segments AA_1 and B_1C_1 meet at point K . The perpendicular bisector of segment A_1K intersects sides AB and AC at L and M , respectively. Prove that points A, A_1, L , and M lie on a circle.