Lectures on Challenging Mathematics

Introduction to Math Olympiads

Geometry

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Zuming Feng
Phillips Exeter Academy and IDEA Math
zfeng@exeter.edu

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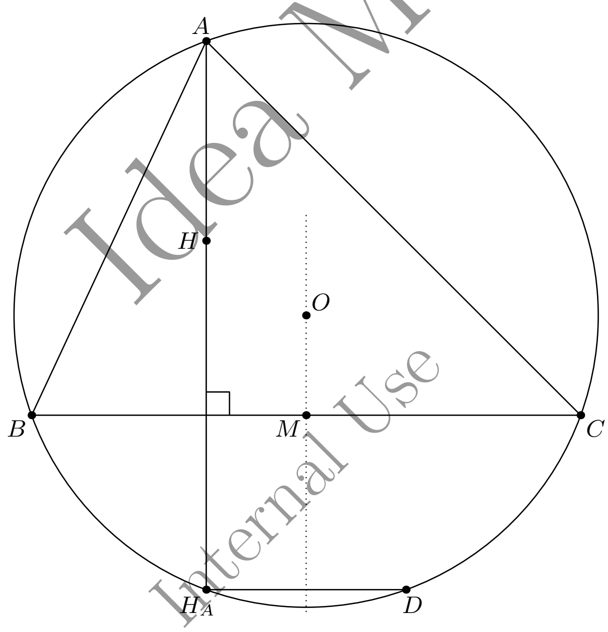
1.12 Angle chasing and the centers of triangles (part 2)

1. The altitudes BE and CF in triangle ABC intersect in point H. Denote by O the circumcenter of triangle ABC. Prove that line AO is perpendicular to line EF.

2. The altitudes AA_1, BB_1, CC_1 of an acute triangle ABC intersect in point H. Show that the orthocenter of triangle ABC, H, is the incenter of triangle $A_1B_1C_1$. What roles do points A, B, C play relative to triangle $A_1B_1C_1$?

3. (Continuation) Suppose ABC is an obtuse triangle with $\angle A > 90°$. Describe the orthocenter of triangle ABC in relation with triangle $A_1B_1C_1$.

4. Let ABC be an acute triangle with circumcircle ω. Let O and H denote its circumcenter and orthocenter. Denote by M be the midpoint of BC. Point H_A lies on $\widehat{BC}$ (not including A) such that $AH_A \perp BC$. Let D be the reflection of H_A across line OM.



Prove that

(a) H and H_A are symmetric across the line BC.

(b) points H, M, and D are collinear.

5. In parallelogram $ABCD$, point H is the orthocenter of triangle ABC. The line through H parallel to line AB meets line BC at P and AD at Q; the line through H parallel to line BC meets line AB at R and CD at S. Prove that P, Q, R, S lie on a circle.

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