Lectures on Challenging Mathematics

Introduction to Math Olympiads

Combinatorics

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1.19 Mathematical arguments – Parity argument

1. Twenty five boys and twenty five girls sit around a table. Prove that it is always possible to find a person both of whose neighbors are girls.

2. (Via Svetoslav Savchev from *Mathematical Miniatures*, by Titu Andreescu and Svetoslav Savchev) A row of minus signs is written on a blackboard. Two players take turns in replacing either a single minus sign by a plus sign or two adjacent minus signs by two plus signs. When a player cannot make a move he or she loses. Can the player who starts force a win?

3. Ten distinct numbers from the set $\{0, 1, 2, \ldots, 14\}$ are to be chosen to fill in the ten circles in the following diagram. The absolute values of the differences of the two numbers joined by each segment must be different from the values for all other segments. Is it possible to do this? Justify your answer.

4. Show that n is divisible by 4 if and only if there are integers $x_1, x_2, \ldots, x_n$ with $|x_1| = |x_2| = \cdots = |x_n|$ such that
$$x_1x_2 + x_2x_3 + \cdots + x_{n-1}x_n + x_nx_1 = 0.$$

5. Find the maximum value of
$$|\cdots|x_1 - x_2| - x_3| - \cdots| - x_{2010}|,$$
where $(x_1, x_2, \ldots, x_{2010})$ is a permutation of $(1, 2, \ldots, 2010)$.