1.4 The sum of divisors

1. For a positive integer n denote by $\sigma(n)$ the sum of its positive divisors, including 1 and n itself. It is clear that
$$\sigma(n) = \sum_{d|n} d.$$
Show that if $n = p_1^{\alpha_1} \cdots p_k^{\alpha_k}$ is the prime factorization of n, then
$$\sigma(n) = \frac{p_1^{\alpha_1+1} - 1}{p_1 - 1} \cdots \frac{p_k^{\alpha_k+1} - 1}{p_k - 1}.$$

2. Find the sum of
   (a) the positive divisors of each of 10000 and 3600;
   (b) the even positive divisors of each of 10000 and 9000;
   (c) the odd positive divisors of each of 6075 and 472500.

3. Compute the sum of all numbers of the form a/b, where a and b are relatively prime positive divisors of 27000.

4. Find the largest divisor of 1001001001 that does not exceed 10000.

5. For a prime p we say that p^k *fully divides* n and write $p^k \| n$ if k is the greatest positive integer such that $p^k \mid n$. Find n such that 2^n fully divides $3^{1024} - 1$.

1.7 Computational practices (part 3)

1. What is the least positive integer that can be expressed as the sum of nine consecutive integers, the sum of ten consecutive integers, and the sum of eleven consecutive integers?

2. How many ordered triples of positive integers (a, b, c) are there for which $a^4b^2c = 54000$?

3. Find all positive integers n that have exactly six positive divisors, including 1 and itself, and the sum of all positive divisors of n is equal to 434.

4. Given that x, y are positive integers with x as small as possible, and y minimized with that constraint, and $x(x+1)$ divides $y(y+1)$, but neither x nor $x+1$ divides either y or $y+1$, find $x^2 + y^2$.

5. Compute $\gcd(3^{10} + 3^6 + 2,\ 3^{15} + 3^{11} + 3^6 + 1)$.