

1.3 More medium level problems from EMC² 2010

1. Evaluate $100 - 1 + 99 - 2 + 98 - 3 + \cdots + 52 - 49 + 51 - 50$.
2. Nick multiplies two consecutive positive integers to get $4^5 - 2^5$. What is the smaller of the two numbers?
3. Some mutant green turkeys and pink elephants are grazing in a field. Mutant green turkeys have six legs and three heads. Pink elephants have 4 legs and 1 head. There are 100 legs and 37 heads in the field. How many animals are grazing?
4. There is pair of similar triangles. One triangle has side lengths 4, 6, and 9. The other triangle has side lengths 8, 12 and x . Find the sum of two possible values of x .
5. If $x^2 + \frac{1}{x^2} = 3$, there are two possible values of $x + \frac{1}{x}$. What is the smaller of the two values?
6. Three flavors (chocolate strawberry, vanilla) of ice cream are sold at Brian's ice cream shop. Brian's friend Zerg gets a coupon for 10 free scoops of ice cream. If the coupon requires Zerg to choose an even number of scoops of each flavor of ice cream, how many ways can he choose his ice cream scoops? (For example, he could have 6 scoops of vanilla and 4 scoops of chocolate. The order in which Zerg eats the scoops does not matter.)
7. David decides he wants to join the West African Drumming Ensemble, and thus he goes to the store and buys three large cylindrical drums. In order to ensure none of the drums drop on the way home, he ties a rope around all of the drums at their mid sections so that each drum is next to the other two. Suppose that each drum has a diameter of 3.5 feet. David needs m feet of rope. Given that $m = a\pi + b$, where a and b are rational numbers, find sum $a + b$.
8. Alice has a rectangular room with 6 outlets lined up on one wall and 6 lamps lined up on the opposite wall. She has 6 distinct power cords (red, blue, green, purple, black, yellow). If the red and green power cords cannot cross, how many ways can she plug in all six lamps?
9. Tracy wants to jump through a line of 12 tiles on the floor by either jumping onto the next block, or jumping onto the block two steps ahead. An example of a path through the 12 tiles may be: 1 step, 2 steps, 2 steps, 2 steps, 1 step, 2 steps, 2 steps. In how many ways can Tracy jump through these 12 tiles?
10. Evaluate

$$\sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \sqrt{1 + \frac{1}{3^2} + \frac{1}{4^2}} + \cdots + \sqrt{1 + \frac{1}{19^2} + \frac{1}{20^2}}.$$