1.16 More on unit circle trigonometry (part 1)

1. Find all x, y, z, w in the interval $[-360°, 360°]$ such that $\cos x = \sin y = \sin 237°$, $\tan z = \tan 237°$, and $\tan w \tan 237° = 1$.

2. Arrange (without calculating devices) the following values from left to right in nondecreasing order:
$$\cos 235°, \quad \cos 253°, \quad \cos 325°, \quad \cos 352°, \quad \cos 523°, \quad \cos 532°$$

3. Find all x and y in the interval $[0°, 360°]$ such that
   (a) $\cos x \geq \cos 527°$ and $\sin y \geq \sin 527°$
   (b) $\cos x \leq \sin 527°$ and $\sin y \leq \cos 527°$

4. Find all x in the interval $[0°, 360°]$ such that $\tan x \geq \tan 432°$.

5. Jamie rides a Ferris wheel for five minutes. The diameter of the wheel is 10 meters, and its center is 6 meters above the ground. Each revolution of the wheel takes 30 seconds. Being more than 9 meters above the ground causes Jamie to suffer an anxiety attack. For how many seconds does Jamie feel uncomfortable?

   What graph is traced by the equation $(x, y) = (5\sin 12t, 6 - 5\cos 12t)$? Think of another equation that will produce the same graph. Use your calculator to check.

1.23 Vector operations in Euclidean geometry (part 2)

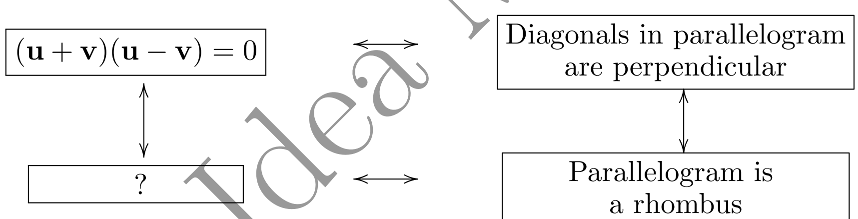
1. In triangle ABC, points M and N are marked on *lines* AB and AC, respectively, so that $\overrightarrow{AM}/\overrightarrow{AB} = \overrightarrow{AN}/\overrightarrow{AC} = r$. Draw a diagram to illustrate each of the following cases.

   (a) $r = 0.7$ (b) $r = 1.6$ (c) $r = -0.4$ (d) $r = -3$

   Show that segments MN and BC are parallel. Explain why this is the SAS similarity theorem.

2. Let $ABCD$ be a trapezoid with $AB \parallel CD$, and let M and N be respective midpoints of sides AD and BC. Set $\overrightarrow{AM} = \mathbf{u}$ and $\overrightarrow{BN} = \mathbf{v}$. Use vector operations to show that $MN \parallel AB \parallel CD$. (Hint: It might be easier to prove something more.)

3. If $\mathbf{u}$ and $\mathbf{v}$ represent the sides of a parallelogram, $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ represent the diagonals. Justify this. Explain what the equation $(\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}-\mathbf{v}) = 0$ tells us about the parallelogram. Give an example of nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ that fit this equation. Use vector operations to complete the following operation diagram.



4. If two vectors $\mathbf{u}$ and $\mathbf{v}$ fit the equation

$$(\mathbf{u}-\mathbf{v})\cdot(\mathbf{u}-\mathbf{v}) = \mathbf{u}\cdot\mathbf{u} + \mathbf{v}\cdot\mathbf{v},$$

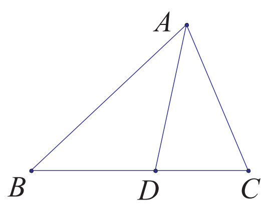
   how must these vectors $\mathbf{u}$ and $\mathbf{v}$ be related? What familiar theorem does this equation represent? Develop a operation diagram for this theorem.

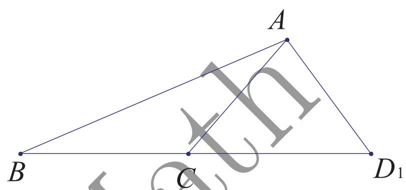
5. Let $\mathbf{u} = [2, -3, 1]$ and $\mathbf{v} = [0, 1, 4]$. Calculate the vector $\mathbf{u}-\mathbf{v}$. Place $\mathbf{u}$ and $\mathbf{v}$ tail-to-tail to form two sides of a triangle. With regard to this triangle, what does $\mathbf{u}-\mathbf{v}$ represent? Calculate the number $\mathbf{u}\cdot\mathbf{u}$ and discuss its relevance to the diagram you have drawn. Do the same for the numbers $\mathbf{v}\cdot\mathbf{v}$ and $(\mathbf{u}-\mathbf{v})\cdot(\mathbf{u}-\mathbf{v})$. How can these numbers help you to find the angles formed by $\mathbf{u}$ and $\mathbf{v}$ (when they are placed tail-by tail)?

1.25 Computations with the laws of sines and the cosines (part 2)

1. [Angle-bisector theorem] The angle-bisector theorem states that:

   Let ABC be a triangle, and let D be a point on segment BC such that $\angle BAD = \angle CAD$. Then
   $$\frac{AB}{AC} = \frac{BD}{CD}.$$





   This theorem can be proved by a synthetic approach. Indeed, we can extend segment BA through A to E such that $CE \parallel AD$ and consider the similar triangles BAD and BEC. We leave these details to the reader as simple exercises.

   Now prove this theorem by using trigonometry.

2. The angle-bisector theorem can be extended to the situation in which AD_1 is the external angle bisector of the triangle. State and prove this result.

3. A parallelogram has a 7-inch side and a 9-inch side, and the longer diagonal is 14 inches long. Find the length of the other diagonal. Do you need your calculator to do it? You should not. Indeed, you should try two approaches without calculator: An approach with the law of cosine and an approach with vector operations.

   This result can be formulated to general result for a parallelogram. One can start with "In a parallelogram, the sum of the squares of lengths of its sides …" Complete this statement and prove it.

4. A triangle has a 5-inch side and an 8-inch side, which form a 60° angle.

   (a) Find the area of this triangle.

   (b) Find the length of the projection of the 8-inch side onto the 5-inch side.

   (c) Find the length of the third side of this triangle.

   (d) Find the sizes of the other two angles of this triangle.

   (e) Find the length of the median drawn to the 8-inch side.

   (f) Find the length of the bisector of the angle opposite the 8-inch side.

   (g) Find the third side of another triangle that has a 5-inch side, an 8-inch side, and the same area as the given triangle.

5. In a triangle, find a relation between the product of its sides, its circumradius, and its area.

1.30 Vector projection (part 2)

1. Vectors $\mathbf{u} = [6, 3, 2]$ and $\mathbf{v} = [7, 24, 60]$ are placed tail-to-tail. Determine the length of projection of $\mathbf{u}$ onto $\mathbf{v}$. Determine the vector projection of $\mathbf{u}$ onto $\mathbf{v}$.

2. Let vectors $\mathbf{u}$ and $\mathbf{v}$ are placed tail-to-tail, and let $\mathbf{w}$ be the vector projection of $\mathbf{v}$ onto $\mathbf{u}$. Explain why that $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$.

3. Let vectors $\mathbf{u}$ and $\mathbf{v}$ form an angle θ when placed tail-to-tail, and let $\mathbf{w}$ be the vector projection of $\mathbf{v}$ onto $\mathbf{u}$. Express $\theta, |\mathbf{w}|$, and $\mathbf{w}$ in term of $\mathbf{u}$ and $\mathbf{v}$.

4. Give two reasons why the projection of $\mathbf{u}$ onto $\mathbf{v}$ is not the same as the projection of $\mathbf{v}$ onto $\mathbf{u}$.

5. Let $A = (0, 0, 0)$, $B = (9, 8, 12)$, and $C = (6, 2, 3)$. Find coordinates for the point on line AB that is closest to C by each of the following three approaches.

   (a) Minimizing a quadratic function.

   (b) Finding two perpendicular vectors.

   (c) Finding the projection of a vector onto another vector.