3.3 Completing the picture

1. The equiangular convex hexagon $ABCDEF$ has $AB = 1$, $BC = 4$, $CD = 2$, and $DE = 4$. Find $[ABCDEF]$.

2. Let $ABCD$ be a square, and let ADE be a triangle constructed outside of the square. Given that $EA = 4$, $AD = 5$, and $DE = 3$, find the distance from E to the center of the square.

3. In quadrilateral $ABCD$, it is given that $\angle A = 120^\circ$, $\angle B = \angle D = 90^\circ$, $AB = 13$, and $AD = 46$. Determine the length of segment AC.

4. Equilateral triangle ABC of side length 2 is drawn. Three squares containing the triangle, $ABDE$, $BCFG$, and $CAHI$, are drawn. What is the area of the smallest triangle that covers these squares?

5. In triangle ABC, $AC = 1$, $\angle BAC = 60^\circ$, $\angle ABC = 100^\circ$. Let E be the midpoint of segment BC, and let D be the point on segment AC such that $\angle DEC = 80^\circ$. Evaluate $[ABC] + 2[CDE]$.

7.3 Solid geometry (part 2)

1. Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

2. Let points $A = (0, 0, 0)$, $B = (1, 0, 0)$, $C = (0, 2, 0)$, and $D = (0, 0, 3)$. Points E, F, G, and H are midpoints of segments BD, AB, AC, and DC respectively. What is the area of quadrilateral $EFGH$?

3. A solid tetrahedron is sliced off a solid wooden unit cube by a plane passing through two nonadjacent vertices on one face and one vertex on the opposite face not adjacent to either of the first two vertices. The tetrahedron is discarded and the remaining portion of the cube is placed on a table with the cut surface facedown. What is the height of this object?

4. Six solid regular tetrahedra are placed on a at surface so that their bases form a regular hexagon $\mathcal{H}$ with side length 1, and so that the vertices not lying in the plane of $\mathcal{H}$ (the *top* vertices) are themselves coplanar. A spherical ball of radius r is placed so that its center is directly above the center of the hexagon. The sphere rests on the tetrahedra so that it is tangent to one edge from each tetrahedron. If the ball's center is coplanar with the top vertices of the tetrahedra, compute r.

5. A unit cube has vertices $P_1, P_2, P_3, P_4, P_1', P_2', P_3'$, and P_4'. Vertices P_2, P_3 , and P_4 are adjacent to P_1, and for $1 \le i \le 4$, vertices P_i and P_i' are opposite to each other. A regular octahedron has one vertex in each of the segments $P_1P_2, P_1P_3, P_1P_4, P_1'P_2', P_1'P_3'$, and $P_1'P_4'$. What is the octahedrons side length?