1.4 PC3M1 practice test 4

1. How many seconds are in a day?
2. Add the algebraic expressions $\frac{5}{x}$ and $\frac{2}{x^2}$. Express your answer in one fraction.
3. What is the quotient and remainder when you divide (a) $x^4 + x^2 - 1$ by $x^2 + 1$? (b) $x^3 + 2x^2 + 5x - 1$ by $x - 2$?
4. If $f(x) = ax^4 - bx^2 + x + 5$ and $f(-3) = 2$, then what is $f(3)$?
5. Bus A is 150 miles due east of Bus B. Both buses start driving due west at constant speeds at the same time. It takes Bus A 10 hours to overtake Bus B. If they had started out at the same time, had driven at the same constant speeds, but had driven toward one another, they would have met in 2 hours. What is the speed, in miles per hour, of Bus A?
6. Evaluate $7 \cdot 11 \cdot 13 \cdot 1003 - 3 \cdot 17 \cdot 59 \cdot 331$.
7. In triangle ABC, $\angle BAC = 45^\circ$ and $\angle ABC = 60^\circ$. Find $\angle ACB$.
8. Determine the least positive integer n such that the following statement is true: Among any n given distinct integers, one can always choose 3 of them such that the sum of the three chosen number is divisible by 3.
9. Let $E(n)$ denote the sum of the even digits of n. For example, $E(5681) = 6 + 8 = 14$. Find $E(1) + E(2) + E(3) + \cdot + E(100)$.
10. Given that $(2x + s)^5 = ax^5 + bx^4 + cx^3 + dx^2 + ex - 243$, compute $a + b + c + d + e$ and $a - b + c - d + e$.

2.5 Selected entry to medium level problems from AMC 2016

1. A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length of 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

2. Trickster Rabbit agrees with Foolish Fox to double Fox's money every time Fox crosses the bridge by Rabbit's house, as long as Fox pays 40 coins in toll to Rabbit after each crossing. The payment is made after the doubling, Fox is excited about his good fortune until he discovers that all his money is gone after crossing the bridge three times. How many coins did Fox have at the beginning?

3. A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to N coins in the Nth row. What is the sum of the digits of N?

4. Laura added two three-digit positive integers. All six digits in these numbers are different. Laura's sum is a three-digit number S. What is the smallest possible value for the sum of the digits of S?

5. At Megapolis Hospital one year, multiple-birth statistics were as follows: Sets of twins, triplets, and quadruplets accounted for 1000 of the babies born. There were four times as many sets of triplets as sets of quadruplets, and there was three times as many sets of twins as sets of triplets. How many of these 1000 babies were in sets of quadruplets?

6. All three vertices of $\triangle ABC$ lie on the parabola defined by $y = x^2$, with A at the origin and $\overline{BC}$ parallel to the x-axis. The area of the triangle is 64. What is the length of BC?

7. Three distinct integers are selected at random between 1 and 2016, inclusive. Let p denote the probability that the product of the three integers is odd. Find the integer n such that $|p - \frac{1}{n}|$ is minimal.

8. Two different numbers are selected at random from $\{1, 2, 3, 4, 5\}$ and multiplied together. What is the probability that the product is even?

9. Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carls garden?

10. How many squares whose sides are parallel to the axes and whose vertices have coordinates that are integers lie entirely within the region bounded by the line $y = \pi x$, the line $y = -0.1$ and the line $x = 5.1$?