1.15 Starry, Starry Night, E15

1. [AMC12A 2012/24] Let $\{a_k\}_{k=1}^{2011}$ be the sequence of real numbers defined by $a_1 = 0.201$, $a_2 = (0.2011)^{a_1}$, $a_3 = (0.20101)^{a_2}$, $a_4 = (0.201011)^{a_3}$, and in general,

$$a_k = \begin{cases} (0.\underbrace{20101\cdots0101}_{k+2 \text{ digits}})^{a_{k-1}} & \text{if } k \text{ is odd,} \\ (0.\underbrace{20101\cdots01011}_{k+2 \text{ digits}})^{a_{k-1}} & \text{if } k \text{ is even.} \end{cases}$$

Rearranging the numbers in the sequence $\{a_k\}_{k=1}^{2011}$ in decreasing order produces a new sequence $\{b_k\}_{k=1}^{2011}$. What is the sum of all integers k, $1 \le k \le 2011$, such that $a_k = b_k$?

Proposed by Zuming Feng

2. [2007 AIME2/3 Modified] Square $ABCD$ has side length 13, and points E and F are exterior to the square so that $BE = DF = 5$ and $AE = CF = 12$. Compute EF.

Proposed by Chris Jeuell

3. [OMO Spring 2018/25] Let m and n be positive integers. Fuming Zeng gives James a rectangle, such that $m-1$ lines are drawn parallel to one pair of sides and $n-1$ lines are drawn parallel to the other pair of sides (with each line distinct and intersecting the interior of the rectangle), thus dividing the rectangle into an $m \times n$ grid of smaller rectangles. Fuming Zeng chooses $m+n-1$ of the mn smaller rectangles and then tells James the area of each of the smaller rectangles. Of the $\binom{mn}{m+n-1}$ possible combinations of rectangles and their areas Fuming Zeng could have given, let $C_{m,n}$ be the number of combinations which would allow James to determine the area of the whole rectangle. Compute

$$\sum_{m+n=100} C_{m,n} \binom{m+n}{m}.$$

Proposed by James Lin