1.13 Starry, Starry Night, E13

1. [AMC10B 2018/23] How many ordered pairs (a, b) of positive integers satisfy the equation

$$a \cdot b + 63 = 20 \cdot \operatorname{lcm}(a, b) + 12 \cdot \gcd(a, b),$$

where $\gcd(a, b)$ denotes the greatest common divisor of a and b, and $\operatorname{lcm}(a, b)$ denotes their least common multiple?

Proposed by Zuming Feng

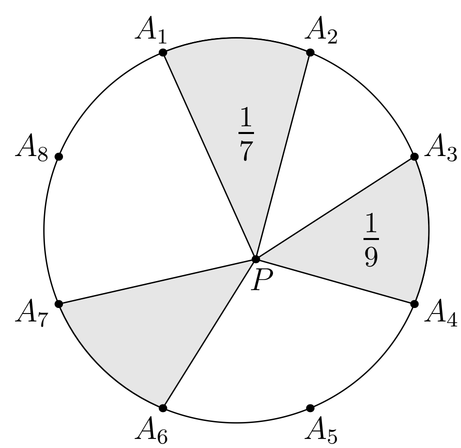
2. [AIME1 2010/7] Define a triple $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ of sets to be *minimally intersecting* if $|\mathcal{A} \cap \mathcal{B}| = |\mathcal{B} \cap \mathcal{C}| = |\mathcal{C} \cap \mathcal{A}| = 1$ and $\mathcal{A} \cap \mathcal{B} \cap \mathcal{C} = \emptyset$. Compute the number of minimally intersecting ordered triples of sets for which each set is a subset of $\{1, 2, 3, 4, 5, 6, 7\}$.

Proposed by Chris Jeuell

3. [ELMO 2016/1] Cookie Monster says a positive integer n is *crunchy* if there exist $2n$ real numbers $x_1, x_2, \ldots, x_{2n}$, not all equal, such that the sum of any n of the x_i's is equal to the product of the other n of the x_i's. Help Cookie Monster determine all crunchy integers.

Proposed by Yannick Yao

4. [AIME2 2019/13] Points A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8 are equally spaced and placed in this order on a circumference of a circle of area 1. Point P lies inside the circle so that the region bounded by segments PA_1, PA_2, and the minor arc $\widehat{A_1A_2}$ of the circle has area $\frac{1}{7}$, while the region bounded by segments PA_3, PA_4, and the minor arc $\widehat{A_3A_4}$ of the circle has area $\frac{1}{9}$. There is a positive integer n such that the area of the region bounded by segments PA_6, PA_7, and the minor arc $\widehat{A_6A_7}$ of the circle is equal to $\frac{1}{8} - \frac{\sqrt{2}}{n}$. Find n.



Proposed by Ivan Borsenco