

## 1.12 Starry, Starry Night, E12

1. [AMC10B 2004/21] Let  $1, 4, \dots$  and  $9, 16, \dots$  be two arithmetic progressions. The set  $S$  is the union of the first 2004 terms of each sequence. How many distinct numbers are in  $S$ ?

Proposed by Zuming Feng

2. [ARML 2016/I6] In triangle  $LEO$ , point  $J$  lies on segment  $LO$  so that  $JE \perp EO$ , and point  $S$  lies on segment  $LE$  so that  $JS \perp LE$ . Given that  $JS = 9$ ,  $EO = 20$ , and  $JO + SE = 37$ , find the perimeter of triangle  $LEO$ .

Proposed by Chris Jeuell

3. [IMO Shortlist 2018/A7] What is the maximum possible value of

$$S = \sqrt[3]{\frac{a}{b+7}} + \sqrt[3]{\frac{b}{c+7}} + \sqrt[3]{\frac{c}{d+7}} + \sqrt[3]{\frac{d}{a+7}}$$

over nonnegative real numbers  $a, b, c, d$  with sum 100?

Proposed by Evan Chen