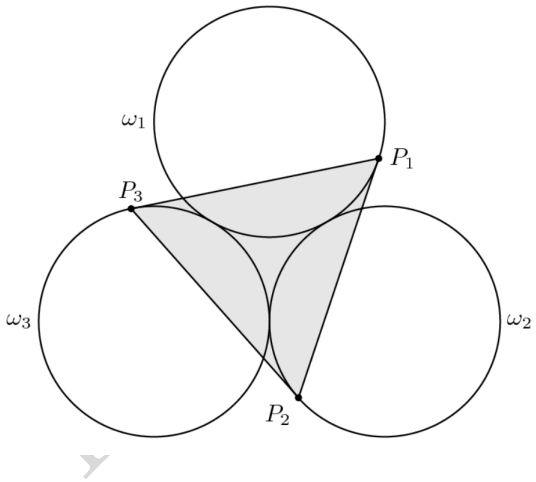
1.11 Starry, Starry Night, E11

1. [AMC12B 2018/15] How many odd positive 3-digit integers are divisible by 3 but do not contain the digit 3?

Proposed by Zuming Feng

2. [AMC12B 2018/25] Circles ω_1, ω_2, and ω_3 each have radius 4 and are placed in the plane so that each circle is externally tangent to the other two. Points P_1, P_2, and P_3 lie on ω_1, ω_2, and ω_3 respectively such that $P_1P_2 = P_2P_3 = P_3P_1$ and line P_iP_{i+1} is tangent to ω_i for each $i = 1, 2, 3$, where $P_4 = P_1$. What is the area of triangle $P_1P_2P_3$?



Proposed by David Joseph Altizio

3. [Canada Repêchage 2019/8] For $t \geq 2$, define $S(t)$ as the number of times t divides into $t!$. We say that a positive integer t is a peak if $S(t) > S(u)$ for all values of $u < t$.

   (a) Prove that there are infinitely many peaks.

   (b) Show that $\lim_{m\to\infty} S(m)/m = 0$.

   (c) Prove or disprove the following statement: For every prime p, there is an integer k for which p divides k and k is a peak.

Proposed by Alex Song