

1.10 Starry, Starry Night, E10

1. [AIME2 2019/3] Find the number of 7-tuples of positive integers (a, b, c, d, e, f, g) that satisfy the following system of equations:

$$abc = 70$$

$$cde = 71$$

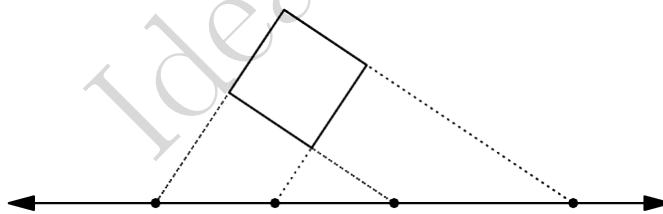
$$efg = 72.$$

Proposed by Chris Jeuell

2. [EMCC 2016/Guts22] Four integers $a, b, c,$ and d with $a \leq b \leq c \leq d$ satisfy the property that the product of any two of them is equal to the sum of the other two. Given that the four numbers are not all equal, determine the 4-tuple (a, b, c, d) .

Proposed by Yannick Yao

3. [AMC12B 2012/17] Square $PQRS$ lies in the first quadrant. Points $(3, 0), (5, 0), (7, 0),$ and $(13, 0)$ lie on lines $SP, RQ, PQ,$ and $SR,$ respectively. What is the sum of the coordinates of the center of the square $PQRS$?



Proposed by Zuming Feng

4. [USATST 2018/3] At a university dinner, there are 2017 mathematicians who each order two distinct entrées, with no two mathematicians ordering the same pair of entrées. The cost of each entrée is equal to the number of mathematicians who ordered it, and the university pays for each mathematician's less expensive entrée (ties broken arbitrarily). Over all possible sets of orders, what is the maximum total amount the university could have paid?

Proposed by Evan Chen