

## 1.9 Starry, Starry Night, E09

1. [IDEAMATH MO1C materials] Find the number of arrangements of five red balls and eleven blue balls in a  $4 \times 4$  table so that each row and each column contains at least one red and one blue ball.

Proposed by Ivan Borsenco

2. [HMMT 2020/Geo7] Let  $\Gamma$  be a circle, and  $\omega_1$  and  $\omega_2$  be two non-intersecting circles inside  $\Gamma$  that are internally tangent to  $\Gamma$  at  $X_1$  and  $X_2$ , respectively. Let one of the common internal tangents of  $\omega_1$  and  $\omega_2$  touch  $\omega_1$  and  $\omega_2$  at  $T_1$  and  $T_2$ , respectively, while intersecting  $\Gamma$  at two points  $A$  and  $B$ . Given that  $2X_1T_1 = X_2T_2$  and that  $\omega_1$ ,  $\omega_2$ , and  $\Gamma$  have radii 2, 3, and 12, respectively, compute the length of  $AB$ .

Proposed by James Lin

3. [CMIMC 2016/N7] Determine the smallest positive prime  $p$  which satisfies the congruence

$$p + p^{-1} \equiv 25 \pmod{143}.$$

Here,  $p^{-1}$  as usual denotes multiplicative inverse.

Proposed by David Joseph Altizio