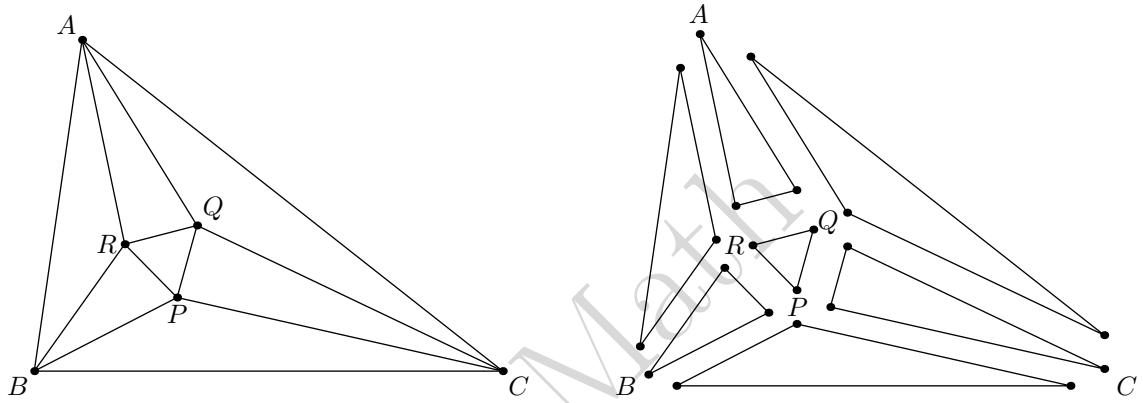
1.8 Starry, Starry Night, E08

1. (Morley's Theorem) In any triangle, the three points of intersection of the adjacent angle trisectors are the vertices of an equilateral triangle.

   We consider the left-hand side diagram shown below. Suppose that $\angle A = 3x$, $\angle B = 3y$, and $\angle C = 3z$. Suppose that Morley's Theorem is true; that is, triangle PQR is equilateral. Express each angle in the diagram in terms of x, y, z.



   For any given triangle ABC, triangle PQR is *uniquely* defined. This fact allows us to work backwards: Start with an equilateral triangle PQR, construct six other triangles (see the right-hand side diagram shown above), and show that they can fit together to form triangle ABC, completing the proof of (the inverse of) Morley's Theorem.

   In memory of John Conway

2. [Canada MO 2020] There are 120 people on a social media platform, where any pair of them may or may not be friends. For any group of 60 people, there are at least 60 pairs of them that are friends. How many friendships, that is, pairs of people that are friends, must there be in total among the 120 people?

   Proposed by Alex Song

3. [Online Math Open, Fall 2013] Let $ABCD$ be a quadrilateral with $AD = 20$ and $BC = 13$. The area of triangle ABC is 338 and the area of triangle DBC is 212. Compute the smallest possible perimeter of $ABCD$.

   Proposed by Evan Chen

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