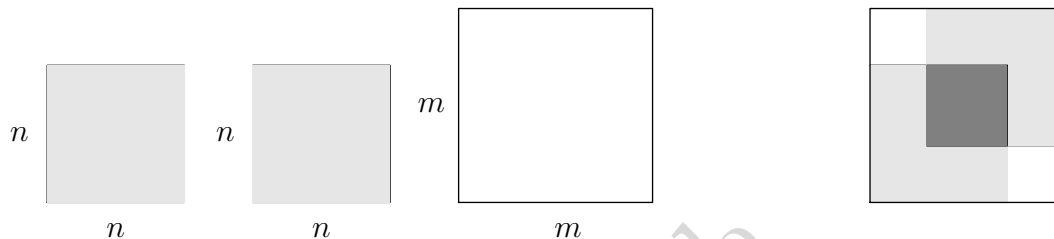


1.7 Starry, Starry Night, E07

1. The famous question: “Is the square root of 2 a ratio of two whole numbers?” can be put another way: “Could there be two squares with side equal to a whole number, n , whose total area is identical to that of a single square with side equal to another whole number, m ?” Explain why and then solve the problem using the diagrams below.



In memory of John Conway

2. [CMIMC 2017/N5] Define the greatest common divisor of two positive rational numbers as follows: for $a, b, c,$ and d positive integers with $\gcd(a, b) = \gcd(c, d) = 1$, write

$$\gcd\left(\frac{a}{b}, \frac{c}{d}\right) = \frac{\gcd(ad, bc)}{bd}.$$

For an integer K , let $f(K)$ denote the number of ordered pairs of rational numbers (m, n) with $0 < m, n < 1$ such that $\gcd(m, n) = \frac{1}{K}$. What is $f(2017) - f(2016)$?

Proposed by David Joseph Altizio

3. [Extension of HMNT 2018/Th10] Each cell in a 300×300 grid of cells are colored red or blue, so that each red cell is adjacent to at least three blue cells and each blue cell is adjacent to at least two red cells. Find
- the minimum possible number of red-blue dominos.
 - the maximum possible number of red cells; characterize all equality cases.
 - the minimum possible number of red cells.

Proposed by James Lin