

1.6 Starry, Starry Night, E06

1. [ARML 2014/I1] Charlie was born in the twentieth century. On his birthday in the year 2014, he notices that his current age is twice the number formed by the rightmost two digits of the year in which he was born. Compute the four-digit year in which Charlie was born.

Proposed by Chris Jeuell

2. [AMC12B 2013/19] In triangle ABC , $AB = 13$, $BC = 14$, and $CA = 15$. Distinct points D , E , and F lie on segments BC , CA , and DE , respectively, such that $AD \perp BC$, $DE \perp AC$, and $AF \perp BF$. Find the length of segment DF .

Proposed by Zuming Feng

3. [HMMT 2018/Guts14] Given that x is a positive real, find the maximum possible value of

$$\sin\left(\tan^{-1}\left(\frac{x}{9}\right) - \tan^{-1}\left(\frac{x}{16}\right)\right).$$

Proposed by Yannick Yao

4. [Extension of HMNT 2018/Th10] Each cell in a 300×300 grid of cells are colored red or blue, so that each red cell is adjacent to at least three blue cells and each blue cell is adjacent to at least two red cells. Find

- (a) the minimum possible number of red-blue dominos.
- (b) the maximum possible number of red cells; characterize all equality cases.
- (c) the minimum possible number of red cells.

Proposed by James Lin