

## 1.5 Starry, Starry Night, E05

- [IMO 1997/2] In triangle  $ABC$  the angle at  $A$  is the smallest. A line through  $A$  meets the circumcircle again at the point  $U$  lying on the arc  $BC$  opposite to  $A$ . The perpendicular bisectors of  $CA$  and  $AB$  meet  $AU$  at  $V$  and  $W$ , respectively, and the lines  $CV$ ,  $BW$  meet at  $T$ . Show that  $AU = TB + TC$ .

Proposed by David Monk, UK

- [CMIMC 2018/A2] Suppose  $x > 1$  is a real number such that  $x + \frac{1}{x} = \sqrt{22}$ . What is  $x^2 - \frac{1}{x^2}$ ?

Proposed by David Altizio

- [USAJMO 2016/2] Prove that there exists a positive integer  $n < 10^6$  such that  $5^n$  has six consecutive zeros in its decimal representation.

Proposed by Evan Chen

- [HMNT 2018/Th10] *One million bucks* (i.e. one million male deer) are in different cells of a  $1000 \times 1000$  grid. The left and right edges of the grid are then glued together, and the top and bottom edges of the grid are glued together, so that the grid forms a doughnut-shaped torus. Furthermore, some of the bucks are *honest bucks*, who always tell the truth, and the remaining bucks are *dishonest bucks*, who never tell the truth. Each of the million bucks claims that “at most one of my neighboring bucks is an honest buck.” A pair of neighboring bucks is said to be *buckaroo* if exactly one of them is an honest buck. What is the minimum possible number of buckaroo pairs in the grid?

Note: Two bucks are considered to be *neighboring* if their cells  $(x_1, y_1)$  and  $(x_2, y_2)$  satisfy either:  $x_1 = x_2$  and  $y_1 - y_2 \equiv \pm 1 \pmod{1000}$ , or  $x_1 - x_2 \equiv \pm 1 \pmod{1000}$  and  $y_1 = y_2$ .

Proposed by James Lin