1.4 Starry, Starry Night, E04

1. Find the greatest five-digit number whose digits have a product of 120.

   Adapted from 2018 AMC 8 #14, Proposed by Chris Jeuell

   What is the largest five-digit integer whose digits have a product equal to 7!? How many five-digit integers are there whose digits have a product equal to 7!?

   IDEA MATH MC3C Materials

2. [AIME2 08/12] There are two distinguishable flagpoles, and there are 19 flags, of which 10 are identical blue flags, and 9 are identical green flags. Find the number of distinguishable arrangements using all of the flags in which each flagpole has at least one flag and no two green flags on either pole are adjacent.

   Proposed by Chris Jeuell

3. Let ABC be a triangle. Circle ω_1 passes through point B and is tangent to line AC at A. Circle ω_2 passes through point C and is tangent to line AB at A. Circles ω_1 and ω_2 intersect at A and P. Ray AP intersects the circumcircle of triangle ABC at A and Q. Prove that $AP = PQ$.

   Extension of 2019 AIME2 #11

4. Prove or disprove: there exist distinct positive integers $a_1, a_2, \ldots, a_n$ such that

$$\frac{1}{a_1^2} + \frac{1}{a_2^2} + \cdots + \frac{1}{a_n^2} = \frac{1}{2}.$$

   Proposed by Alex Song