

# Chapter 1

## Starry, Starry Night, Spring 2020

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### 1.1 Starry, Starry Night, E1

1. We are given a rectangle. Suppose two perpendicular lines, each parallel to one of the rectangle's sides divide the rectangle into four smaller rectangles. Given that the areas of three of the smaller rectangles are equal to 4, 6, and 8, find the least possible perimeter of the original rectangle.

Proposed by Ivan Borsenco

2. [HMMT 20/4] Given an  $8 \times 8$  checkerboard with alternating white and black squares, how many ways are there to choose four black squares and four white squares so that no two of the eight chosen squares are in the same row or column?

Proposed by James Lin

3. [AIME1 04/9] Let  $ABC$  be a triangle with sides 3, 4, and 5, and  $DEFG$  be a 6-by-7 rectangle. A segment is drawn to divide triangle  $ABC$  into a triangle  $U_1$  and a trapezoid  $V_1$ , and another segment is drawn to divide rectangle  $DEFG$  into a triangle  $U_2$  and a trapezoid  $V_2$  such that  $U_1$  is similar to  $U_2$  and  $V_1$  is similar to  $V_2$ . Find the minimum value of the area of  $U_1$ .

Proposed by Zuming Feng

4. [EMCC 2015/G22] Define a sequence of positive integers  $s_1, s_2, \dots, s_{10}$  to be *terrible* if the following conditions are satisfied for any pair of positive integers  $i$  and  $j$  satisfying  $1 \leq i < j \leq 10$ :  $s_i > s_j$  and  $j - i + 1$  divides the quantity  $s_i + s_{i+1} + \dots + s_j$ . Determine the minimum possible value of  $s_1 + s_2 + \dots + s_{10}$  over all terrible sequences.

Proposed by Yannick Yao