1.8 Season 1 Episode 8, 11/8/2015

1. You are presented with two fuses (lengths of string), each of which will burn for exactly 1 minute, but not uniformly along its length. Can you use them to measure 45 seconds?

2. [AMC10A 2013] Let points $A = (0, 0)$, $B = (1, 2)$, $C = (3, 3)$, and $D = (4, 0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects CD at point $\left(\frac{p}{q}, \frac{r}{s}\right)$, where these fractions are in lowest terms. What is $p + q + r + s$?

3. The numbers $1, 2, \ldots, 8$ are placed in the 3×3 grid, leaving exactly one blank square. (The blank square can be any square in the grid). Such a placement is called *okay* if in every pair of adjacent squares, either one square is blank or the difference between the two numbers is at most 2 (two squares are considered adjacent if they share a common side). (The placement shown on the right-hand side is not okay because 1 and 4 are placed in two neighboring squares.) If reflections, rotations, etc. of placements are considered distinct, compute the number of distinct okay placements.

2		3
1	4	5
6	8	7

4. On each of 10 sheets of paper are written several (not necessarily distinct) powers of 2. The sum of the numbers on each sheet is the same. Assume that each power of 2 was written at most M times. Determine the minimum value of M.

5. [IMO 2015, by Merlijn Staps from Netherlands] We say that a finite set $\mathcal{S}$ of points in the plane is *balanced* if, for any two distinct points A and B in $\mathcal{S}$, there is a point C in $\mathcal{S}$ such that $AC = BC$. We say that $\mathcal{S}$ is *center-free* if for any distinct points A, B, and C in $\mathcal{S}$, there is no point P in $\mathcal{S}$ such that $PA = PB = PC$. Find all positive integers $n \geq 3$ satisfy the following properties: There exist a balanced set consisting of n points and there does not exist a balanced, center-free set consisting of n points.