1.5 Season 1 Episode 5, 10/18/2015

1. Let $A = (0, 0)$, $B = (12, 9)$, $C = (2, 4)$, $P = (0, 4)$, and $Q = (-2, 4)$. Decide whether angles BAC and PAQ are the same size (congruent, that is), and give your reasons. (Trigonometry method shall not be used in your solution.)

2. [Putnam 2004] Basketball star Shanille OKeals team statistician keeps track of the number, $S(N)$, of successful free throws she has made in her first N attempts of the season. Early in the season, $S(N)$ was less than 80% of N, but by the end of the season, $S(N)$ was more than 80% of N. Was there necessarily a moment in between when $S(N)$ was exactly 80% of N?

3. [AIME1 2004, by Zuming Feng] Let ABC be a triangle with sides $3, 4$, and 5, and $DEFG$ be a 6-by-7 rectangle. A segment is drawn to divide triangle ABC into a triangle U_1 and a trapezoid V_1, and another segment is drawn to divide rectangle $DEFG$ into a triangle U_2 and a trapezoid V_2 such that U_1 is similar to U_2 and V_1 is similar to V_2. The minimum value of the area of U_1 can be written in the form m/n, where m and n are relatively prime positive integers. Find $m + n$?

4. [Ideamath San Jose Summer Program test, By Matthew Superdock] A robot moves around a plane tiled by equilateral triangles of unit side length. (Each triangle shares a side with three other triangles.) He begins at a vertex of one of the triangles and moves along sides of the triangles. How many paths of length 5 can he take such that after traveling 5 units, he is back at his starting position?

5. [USAMO 2004, by Ricky Liu] Let k be a real number greater than 1. Show that it is possible to dissect a $1 \times k$ rectangle into two similar, but incongruent, polygons?